DUE: TUESDAY FEBRUARY 1, 2011

To receive full credit, you must exhibit the intermediate steps that lead you to your final results.

1. The series for the principal value of the complex-valued logarithm,

$$\operatorname{Ln}(1-z) = -\sum_{n=1}^{\infty} \frac{z^n}{n}\,, \tag{1}$$

converges for all $|z| \leq 1$, $z \neq 1$. In particular, consider the conditionally convergent series,

$$S \equiv \sum_{n=1}^{\infty} \frac{e^{in\theta}}{n}\,, \qquad \text{where } 0 < \theta < 2\pi\,. \tag{2}$$

(a) By taking the real part of eq. (2), evaluate

$$\sum_{n=1}^{\infty} \frac{\cos n\theta}{n}\,, \qquad \text{where } 0 < \theta < 2\pi\,,$$

as a function of θ. Check that your answer has the right limit for $\theta = \pi$.

(b) By taking the imaginary part of eq. (2), prove that

$$\sum_{n=1}^{\infty} \frac{\sin n\theta}{n} = \tfrac{1}{2}(\pi - \theta)\,, \qquad \text{where } 0 < \theta < 2\pi\,.$$

<u>*HINT*:</u> The simplest method for solving this problem is to first sum the series given in eq. (2) [using the result of eq. (1)]. Then, simplify the resulting expression for the complex-valued logarithm by making use of one of the two methods employed to solve problem 4(c) of the midterm, which is described in detail on the midterm exam solutions. Finally, take the real and imaginary parts of the final result to solve parts (a) and (b) above.

2. Evaluate the integral

$$I_n = \int_0^{\infty} t^n e^{-kt^2}\, dt\,,$$

for $k > 0$ and $n > -1$. [*HINT:* By a suitable change of variables, show that I_n can be expressed in terms of the defining integral for the Gamma function.]

3. Boas, p. 540, problem 11.3–5.

4. Boas, p. 540, problem 11.3–13.

5. Boas, p. 542, problem 11.5–3.

NOTE: There is a missing left parenthesis in the problem. You are asked you to prove that

$$\binom{p}{n} = \frac{\Gamma(p+1)}{n!\,\Gamma(p-n+1)}\,.$$

6. Boas, p. 542, problem 11.5–5, part (a)