1. Consider the real valued function:

$$g(x) = \left(\frac{3}{x^3} - \frac{1}{x}\right)\sin x - \frac{3}{x^2}\cos x\,.$$

(a) Compute $\lim_{x\to 0} g(x)$.

(b) Find the *behavior* of $g(x)$ as $x \to 0$.

2. Consider the real-valued function:

$$f(x) = \tfrac{1}{2}\ln\left(\frac{1+x}{1-x}\right)\,.$$

(a) Derive the Taylor series expansion of $f(x)$ about the point $x = 0$. Write the series using summation notation (that is, you will need to determine the general term in the series).

HINT: You may use any Taylor series previously obtained in Boas as a starting point.

(b) Determine all possible values of x for which the series obtained in part (a) converges.

(c) Evaluate explicitly the sum

$$\sum_{n=0}^{\infty} \frac{1}{2^{2n}}\,\frac{1}{2n+1}\,.$$

Use your calculator to compute the sum of the first four terms of the series, and compare this numerical approximation with the exact result.

HINT: You may find the results of part (a) helpful in this regard.

3. Evaluate the following quantities. If complex, express the quantity in $x + iy$ form. If the quantity is multi-valued, you should provide all possible values.

(a) 1^π

(b) $\mathrm{Arg}(\sin i)$

(c) $\mathrm{Im}\ \ln(i-1)$

4. Assume that p is a real parameter such that $-1 < p < 1$.

(a) Compute the following sum:

$$\sum_{n=0}^{\infty} p^n e^{in\theta} .$$

(b) Using the results of part (a), compute the sum

$$\sum_{n=0}^{\infty} p^n \cos(n\theta) .$$

Verify that your result for the sum in part (b) has the correct form in the $\theta \to 0$ limit.