Here is a collection of practice problems suitable for the first midterm exam.

1. Evaluate the following limits:

(a) $\displaystyle\lim_{x\to 0}\left(\frac{1+x}{x}-\frac{1}{\sin x}\right)\,,$

(b) $\displaystyle\lim_{n\to\infty}\sqrt{n^2+3n}-n\,,$

(c) $\displaystyle\lim_{x\to\infty}\frac{\ln x}{\sqrt{x}}\,.$

2. Find the radius of convergence of the following three series:

$$\text{(a)}\ \sum_{n=1}^{\infty}\frac{x^n}{\ln(n+1)}\,,\qquad \text{(b)}\ \sum_{n=0}^{\infty}\frac{(n!)^2x^n}{(2n)!}\,,\qquad \text{(c)}\ \sum_{n=0}^{\infty}\frac{n^2(x-5)^n}{5^n(n^2+1)}\,.$$

3. Determine whether the following series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=2}^{\infty}\frac{(-1)^n}{n^2-n}\,.$$

If this series is convergent, determine its sum.

4. Determine whether the following series are absolutely convergent, conditionally convergent or divergent.

$$\text{(a)}\ \sum_{n=1}^{\infty}(-1)^n\frac{n}{n+1}\,,\qquad \text{(b)}\ \sum_{n=1}^{\infty}\frac{(-1)^n}{2^{\ln n}}\,,\qquad \text{(c)}\ \sum_{n=2}^{\infty}\frac{1}{n\ln n}\,.$$

5. What is the *behavior* of the function:

$$f(x)=-1+\frac{1}{x^2}\left[\frac{1}{(1+x^2)^{3/2}}-\frac{1}{(1+x^2)^{5/2}}\right]\,,$$

as $x\to 0$? (Obtaining the limit as $x\to 0$ is not sufficient.)

6. Evaluate $f(x)=\ln\sqrt{(1+x)/(1-x)}-\tan x$ at $x=0.0015$ without a calculator. Determine the numerical accuracy of your result. Is your calculator a useful tool for this problem? (Try it!)

7. For each expression find all possible values and express your result both in the form $x + iy$ and in polar form $re^{i\theta}$, where θ is the principal value of the argument.

(a) $i^{77} + i^{202}$ (b) $\dfrac{3+i}{2+i}$ (c) $\sqrt{-2 + 2i\sqrt{3}}$

(d) $\left(\dfrac{1+i}{1-i}\right)^4$ (e) $\sqrt[4]{16}$

8. Let $z = 1 - i$. Express each of the following in the form of $x + iy$. For any multi-valued function, you should indicate all possible values of the result.

(a) $\cos(1/z)$ (b) z^z (c) $\tan(z-1)$

(d) $\mathrm{Ln}\, z$ (e) $\arg z$

9. Solve for all possible values of the real numbers x and y in the following equations:

(a) $x + iy = y + ix$.

(b) $\dfrac{x+iy}{x-iy} = -i$.

10. Find the disk of convergence of the following complex power series:

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}\, z^n$, (b) $\displaystyle\sum_{n=1}^{\infty} \frac{z^{2n}}{(2n+1)!}$.

11. Evaluate the integral

$$\int_0^{\pi} \sin 3x \cos 4x \, dx \, .$$

HINT: Rewrite the trigonometric functions in exponential form.

12. Evaluate the following quantities:

(a) $(-1)^i$

(b) $\mathrm{Im}\, [ix + \sqrt{1-x^2}]^{-1}$, where x is a real number and $|x| < 1$

(c) $\arg(e^{x+iy})$, where x and y are real numbers

Be sure to indicate all possible values if the quantity in question is multi-valued. Simplify your expressions as much as possible.