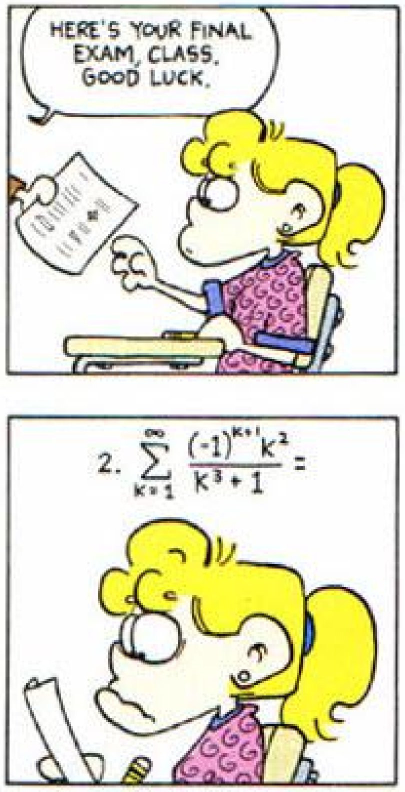
Physics 116A **First Midterm Exam** **Winter 2011**

INSTRUCTIONS: This is a one-hour exam. During the exam, you may refer to the textbook (Boas), the class handouts (including solution sets to the homeworks and the practice problems and exam) or your own personal notes. Collaboration with your neighbor is strictly forbidden.

In answering the questions, it is not sufficient to simply give a final result. You must provide the intermediate steps needed to arrive at the final solution in order to get full credit. *However, if you are employing any result already obtained in class or in the textbook, you do not need to re-derive it.* The point value of each problem is indicated in brackets. The total number of points available is 100, plus bonus points for the extra credit.

1. [30]



(a) In *The Best of Foxtrot*, Volume 1, cartoonist Bill Amend (a physics major from Amherst College), admitted that he made up this problem but it turned out to be "crazy hard." So I will be more forgiving than Paige's professor and simply ask you whether the series

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}k^2}{k^3+1}$$

converges absolutely, converges conditionally or diverges. Explain the reasoning behind your conclusion.

(b) Find the interval of convergence of the following power series:

$$g(x) \equiv \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)n!}\,,$$

where x is a real variable.

(c) Using part (b), evaluate $\dfrac{dg}{dx}$ and sum the resulting series to obtain a known elementary function.

2. [20] Consider the real-valued function,

$$f(x) = \frac{1}{x^3}\left[1 + ax - 2\ln(1+ax) - \frac{1}{1+ax}\right],$$

where a is a non-zero finite constant. Find the *behavior* of $f(x)$ as $x \to 0$. (Obtaining the limit of $f(x)$ as $x \to 0$ is not sufficient.)

3. [20] Consider the complex number $w = e^{2\pi i/n}$, where n is an integer larger than 2.

(a) Find the sum of the following finite series,

$$\sum_{k=1}^{n} w^{2k} = w^2 + w^4 + w^6 + \cdots + w^{2n}.$$

(b) What is the value of the sum in part (a) for the special cases of $n = 1$ and $n = 2$?

4. [30] Evaluate the following quantities. If complex, express the quantity in $x + iy$ form. If the quantity is multi-valued, you should provide all possible values.

(a) $(-e)^{i\pi}$

(b) $\cos\left[2i\ln\left(\frac{1-i}{1+i}\right)\right]$

(c) $\mathrm{Arg}(1 + e^{i\theta})$ as a function of θ, where $-\pi < \theta < \pi$.

EXTRA CREDIT [10]: Using the result of part (c) above, identify the curve in the complex plane described by the equation $z = 1 + e^{i\theta}$ as θ varies over its allowed range.