DUE: THURSDAY OCTOBER 18, 2001

1. (a) The line element of special relativity is given by $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$. Transform this line element from the usual $(ct; x, y, z)$ rectangular coordinates to new coordinates $(ct'; x', y', z')$ related by

$$t = \left(\frac{c}{g} + \frac{x'}{c}\right) \sinh\left(\frac{gt'}{c}\right) ,$$

$$x = c\left(\frac{c}{g} + \frac{x'}{c}\right) \cosh\left(\frac{gt'}{c}\right) - \frac{c^2}{g} ,$$

$$y = y' ,$$

$$z = z' ,$$

for a constant g with the dimensions of acceleration.

(b) For $gt'/c \ll 1$, show that this corresponds to a transformation to a uniformly accelerated frame in Newtonian mechanics.

(c) Show that an at-rest clock in this frame at $x' = h$ runs fast compared to a clock at rest at $x' = 0$ by a factor of $(1 + gh/c^2)$. How is this related to the equivalence principle?

2. An atomic clock is placed in the basement of the Empire State Building, and another one is placed on the 102nd floor, 380 meters above the first clock.

(a) Which one runs faster because of the gravitational potential difference between them?

(b) How long will the clocks have to run before one of them gains 1 ns compared to the other because of the gravitational time dilation?

3. An atomic clock is located in an inertial frame on the axis of a centrifuge, and an identical clock is located on the rim of the centrifuge. The linear speed of the rim of the centrifuge is v.

(a) Which clock runs faster?

(b) Assuming $v \ll c$, use the principle of equivalence to determine the fractional difference between the rates of the clocks.

HINT: Replace the centrifugal acceleration by an equivalent gravitational field.

4. Consider a torus (*e.g.*, a donut or bagel) with inner radius of 2 cm and an outer radius of 5 cm (so that the circular cross section through the center of the torus has a radius of 1.5 cm).

(a) Calculate the Gaussian curvature K at a point on the outer edge of the torus.

(b) Calculate the Gaussian curvature K at a point on the inner edge of the torus.

(c) Where on the torus does K vanish?

5. The Schwarzschild metric is given by:

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2 \, .$$

What is the coordinate velocity of light in the Schwarzschild metric as a function of r

(a) in the radial direction?

(b) in the transverse direction?

What are the physical consequences of these results?