DUE: TUESDAY DECEMBER 4, 2001

FINAL EXAM ALERT: The final exam will be an in-class exam, and will take place in 289 Kerr Hall from 8–11 am on Wednesday December 5, 2001. The exam will cover the entire course material. While working on the exam, you are permitted to consult with your class notes, all class handouts (including problem set solutions) and the course textbook by Kenyon. Please bring a calculator to the exam. You should *not* collaborate with anyone or any other source material during the exam.

There will be a special class session held on Tuesday December 4, 2001 in Kerr Hall, Room 289 from 11 am–1 pm. This session will consist of either a course review and/or a special topics lecture. Problem Set #5 must be handed in during this session. Solution Set #5 will be available at the end of the Tuesday session.

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1. The continuity equation for the universe, considered to be a perfect fluid, is given by:

$$\frac{d(\rho R^3)}{dR} = \frac{-3pR^2}{c^2}\,,$$

where $R(t)$ is the scale factor of the universe.

(a) Derive this equation by generalizing the derivation presented on p. 157 of Kenyon (which is appropriate for the case of zero pressure).

HINT: Multiply eq. (11.8) of Kenyon by R^3 and differentiate with respect to time. Then make use of eq. (11.9).

(b) Assuming the that the equation of state $\rho = 3p/c^2$ is satisfied,* show that the continuity equation above implies that $\rho \propto R^{-4}$.

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*This is the equation of state appropriate for radiation.

2. (a) Show that the distance to our particle horizon at time t_0 is

$$d_H = c \int_0^1 \frac{da}{a^2 H},$$

where t_0 corresponds to the value of the cosmic time today and H is the Hubble parameter corresponding to the scale factor $a(t) \equiv R(t)/R(t_0)$.

(b) Show that $d_H = 2ct_H$ for the (matter-dominated) Einstein-de Sitter cosmology (*i.e.*, $\Omega_m = 1$ and $\Omega_\Lambda = 0$). Here, $t_H \equiv H_0^{-1}$, where H_0 is the present day Hubble constant.

(c) How would the result of part (b) change for a radiation-dominated universe? That is, assume $\Omega_m = \Omega_\Lambda = 0$, and the total energy density of the universe due to the radiation is equal to the critical density ($\Omega_r = 1$).

3. Suppose that a galaxy is observed to have a redshift $z = 1$. Assuming a (matter-dominated) Einstein-de Sitter cosmology, at what fraction t/t_0 of the *present* age of the universe did light leave this galaxy?

4. Consider an open universe ($\Omega_T \equiv \Omega_m + \Omega_\Lambda \leq 1$).

(a) Show that if $\Omega_\Lambda = 0$, all of the universe will eventually come within our horizon.

(b) Show that if $\Omega_\Lambda > 0$, the horizon will approach a finite limit. That is, this universe possesses an *event horizon* beyond which we will never see.

5. (a) Consider a matter-dominated flat universe with $\Lambda \neq 0$. That is, the universe is characterized by Ω_m and Ω_Λ, where $\Omega_m + \Omega_\Lambda = 1$. Find an expression for the age of the universe, t_0, in terms of an integral over the scale parameter $a(t) \equiv R(t)/R(t_0)$. Evaluate the integral explicitly.

HINT: The integral can be evaluated by a suitable change of variables. One possible choice is $x = 1/a^3$. Find the resulting integral in your favorite integral table.

(b) Suppose that $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $h = 0.7$, where the present day Hubble constant is given by $H_0 = (100 \text{ km s}^{-1} \text{ Mpc}^{-1})h$. What is the value of t_0?