This is a take home exam. You may refer to the textbook by Jackson, the class handouts and solution sets, and any other material posted to the Physics 214 website. Any reference for integrals or other mathematical facts, and any personal handwritten notes are also OK. However, you should *not* collaborate with anyone else. The point value of each problem is indicated in the square brackets below; use these values as a guide to manage your time during the exam.

Completed exams should be delivered to my ISB campus mailbox (or sent to me via email) by the end of the day on Friday February 17, 2023. Please limit your work to a 24 hour time period starting from when you began the exam.

In order to earn total credit for a problem solution, you must show all work involved in obtaining the solution. There is no need to rederive results that have been previously obtained in the textbook, the class notes or the class handouts. However, if you make use of any previously derived result, please cite the source of the result.

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1. [30] A transmission line consisting of two concentric circular cylinders of metal with conductivity σ and skin depth δ, as shown in Figure 1, is filled with a uniform lossless dielectric with electric permittivity ϵ and magnetic permeability μ. A TEM mode is propagated along this line. Denote the peak value of the azimuthal magnetic field at the surface of the inner conductor by H_0.

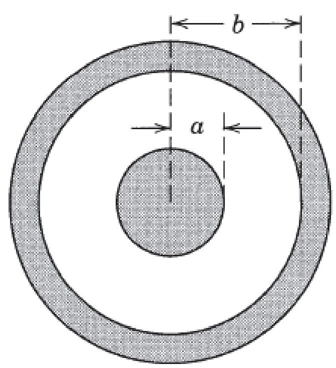

Figure 1: A transmission line consisting of two concentric circular cylinders of metal with inner radius a and outer radius b.

(a) Compute the time-averaged power flow along the line in terms of H_0, a, b, ϵ, and μ.

(b) The transmitted power is attenuated along the line has the form

$$P(z) = P_0 e^{-2\gamma z}\,.$$

Determine the explicit expression for γ in terms of the conductivity σ, the skin depth δ, and the parameters a, b, ϵ, μ.

2. [30] A classical point magnetic dipole moment $\vec{\boldsymbol{\mu}}$ at rest has a vector potential (in gaussian units), which is given by

$$\vec{\boldsymbol{A}} = \frac{\vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{r}}}{r^3}$$

and no scalar potential ($\Phi = 0$). Suppose that the point magnetic dipole moment $\vec{\boldsymbol{\mu}}$ now moves with velocity $\vec{\boldsymbol{v}}$.

(a) Compute and compare the interaction energy between the moving magnetic dipole and static external fields $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ and the interaction energy computed in the rest frame of the magnetic moment. Explain why the two results agree (or disagree). Assume that $v \ll c$ and keep only terms up to $\mathcal{O}(v/c)$.

HINT: Recall eq. (4.24), eq. (5.72) and the results of problem (11.27) of Jackson.

(b) Compute the *exact* expression for the scalar potential generated by a point magnetic dipole $\vec{\boldsymbol{\mu}}$ moving with velocity $\vec{\boldsymbol{v}}$. (Do *not* assume that $v \ll c$.) Express your answer in terms of the angle between the directions of $\hat{\boldsymbol{n}}$ and $\vec{\boldsymbol{v}}$, where $\hat{\boldsymbol{n}}$ is a unit vector pointing from the magnetic dipole to the point at which the scalar potential is measured.

3. [40] A magnetic dipole $\vec{\boldsymbol{m}}$ undergoes precessional motion with angular frequency ω and angle ϑ_0 with respect to the z-axis as shown below. That is, the time-dependence of the azimuthal angle is $\varphi_0(t) = \varphi_0 - \omega t$. Electromagnetic radiation is emitted by the precessing dipole.[1]

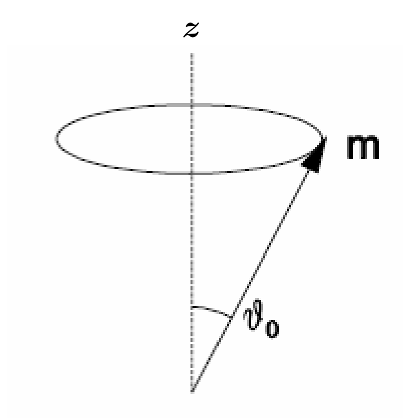

Figure 2: A magnetic dipole $\vec{\boldsymbol{m}}$ undergoes precessional motion with angular frequency ω and angle ϑ_0 with respect to the z-axis.

(a) Write out an explicit expression for the time-dependent magnetic dipole vector $\vec{\boldsymbol{m}}$ in terms of its magnitude m_0, the angles ϑ_0 and φ_0 and the time t. Show that $\vec{\boldsymbol{m}}$ consists of the sum of a time-dependent term and a time-independent term. Verify that the time-dependent term can be written as $\mathrm{Re}(\vec{\boldsymbol{\mu}}\, e^{-i\omega t})$, for some suitably chosen complex vector $\vec{\boldsymbol{\mu}}$.

[1] Radiation from pulsars is believed to be due to this mechanism.

(b) Compute the angular distribution of the time-averaged radiated power, with respect to the z-axis defined in Fig. 2.

(c) Compute the total power radiated.

(d) What is the polarization of the radiation measured by an observer located along the positive z-axis far from the precessing dipole? How would your answer change if the observer were located in the x–y plane?