

INSTRUCTIONS: This is a closed book exam. You may consult one (two-sided) 8 1/2"  $\times$  11" sheet of paper of personal notes. However, you may not collaborate and/or plagiarize anyone else's work. Failure to comply with these rules will result in a zero on this exam.

The use of calculators for the numerical work is strongly encouraged. You have one hour and ten minutes to complete this exam (unless special prior arrangements have been made). The point count of each problem is specified below; use this information to manage your time wisely.

### **PART I: Multiple choice questions**

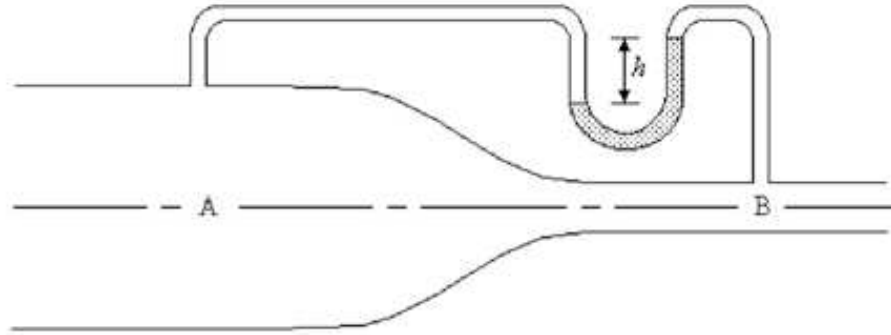
Only one of the choices given is the correct answer. No explanation for your choice is required. Each multiple choice problem is worth 5 points.

1. Start with a glass cup containing an ice cube resting at the bottom of the glass. The cup is now filled with water until the water level reaches the brim of the glass. The tip of the ice cube is initially observed to (partially) stick out of the surface of the water. As the ice cube melts,

- (a) the cup overflows.
- (b) the cup might overflow but it depends on the actual mass of the ice cube.
- (c) the water level remains the same.
- (d) the water level goes down.

2. In simple harmonic motion, the speed is greatest at that point in the cycle when

- (a) the magnitude of the acceleration is a maximum.
- (b) the displacement is a maximum.
- (c) the magnitude of the acceleration is a minimum.
- (d) the potential energy is a maximum.
- (e) the kinetic energy is a minimum.



3. Water flows in a horizontal pipe as shown in the figure above. The cross-sectional area of the pipe at  $A$  is larger than the cross-sectional area at  $B$ . The shaded area indicated in the figure is mercury, and the unshaded area between the horizontal pipe containing the flowing water and the mercury is occupied by air. According to the above figure, the height of the mercury on the right side of the U-tube is higher than the height of the mercury on the left side of the U-tube. Is this correct?

- (a) Yes, the relative heights of mercury on the two sides as shown above is correct.
- (b) No, the relative heights of mercury on the two sides should be equal.
- (c) No, the relative height of the mercury on the right should be lower than the height of the mercury on the left.
- (d) There is insufficient information to answer this question definitively.

4. If damping causes the amplitude of oscillation of a system to decrease by 50%, then the ratio of the final energy to the initial energy of the system is:

- (a) 1
- (b)  $1/\sqrt{2}$
- (c)  $1/2$
- (d)  $1/4$

5. Two underdamped oscillators are known to have the same natural frequency  $\omega_0$ . The mass and damping coefficient of the first oscillator are  $m_1$  and  $b_1$ , and the mass and damping coefficient of the second oscillator are  $m_2$  and  $b_2$ , respectively. A sinusoidal driving force of  $F_{\text{ext}} = F_0 \cos \omega t$  is applied to each oscillator. Starting with  $\omega$  far from  $\omega_0$ , the driving force is tuned in order to observe resonant behavior. If  $m_1 = 4m_2$  and  $b_1 = 2b_2$ , then which one of the following statements concerning the resonant amplitude of the driven oscillations is correct?

(a) The resonant peak of the first driven oscillator is higher and narrower than that of the second oscillator.

(b) The resonant peak of the first driven oscillator is higher and wider than that of the second oscillator.

(c) The resonant peak of the first driven oscillator is lower and wider than that of the second oscillator.

(d) The resonant peak of the first driven oscillator is lower and narrower than that of the second oscillator.

(e) The resonant peak of both driven oscillators possess the same characteristics.

6. Two strings, one thick and the other thin, are connected to form one long string of constant tension. A wave travels along the string and passes the point where the two strings are connected. When the wave passes from the thick string to the thin string, which one of the following statements concerning the period  $T$ , the frequency  $f$  and the wavelength  $\lambda$  of the wave is correct?

(a) only the period changes

(b) only the frequency changes

(c) only the wavelength changes

(d) the period and frequency both change

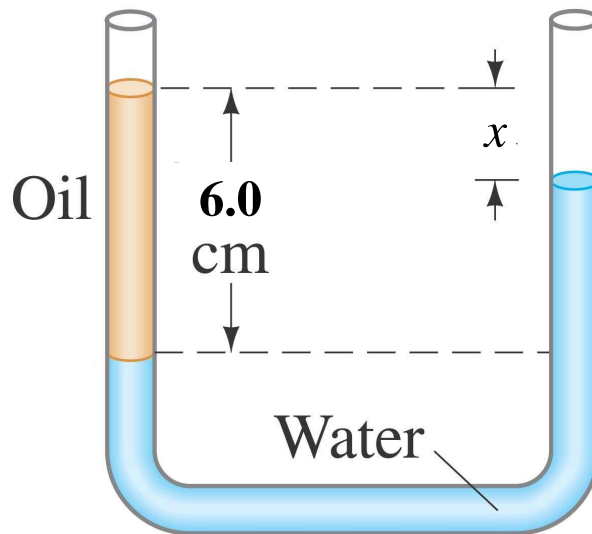
(e) all three quantities  $T$ ,  $f$  and  $\lambda$  change

(f) none of the three quantities  $T$ ,  $f$  and  $\lambda$  change

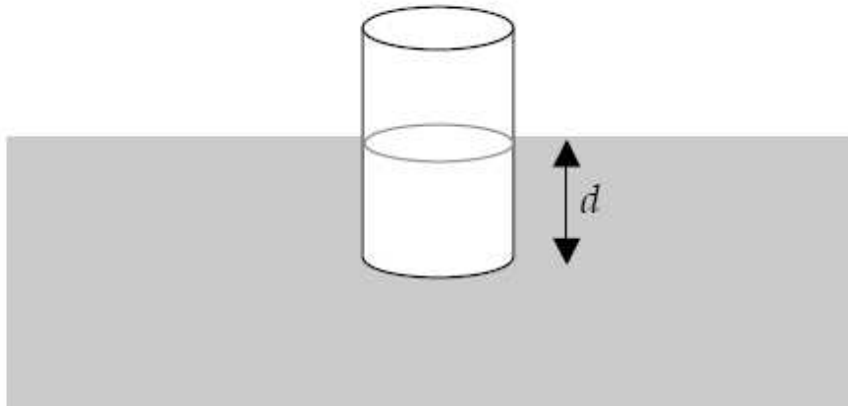
## PART II: Short problems

To earn full credit on the following problems, you must exhibit the steps that lead to your final result (and will depend on the clarity of your method of solution as well as on your final answer). Problem 7 and 9 are worth 20 points each, and problem 8 is worth 30 points.

7. A U-tube is open to the atmosphere at both ends. After water (with density  $10^3 \text{ kg/m}^3$ ) is poured into the tube, the heights of the two water columns on the vertical sides of the U are equal. Oil, with a density of  $950 \text{ kg/m}^3$ , is then poured into one side of the U-tube, until the column of oil (which floats on top of the water) is  $6.0 \text{ cm}$  tall, as shown in the figure below. How much higher is the top surface of the oil as compared with the surface of the water on the other side of the tube (this quantity is denoted by  $x$  in the figure below)?



8. An empty steel drum of mass  $M$ , height  $h$  and cross-sectional area  $A$  is floating nearly submerged in the water as depicted below.



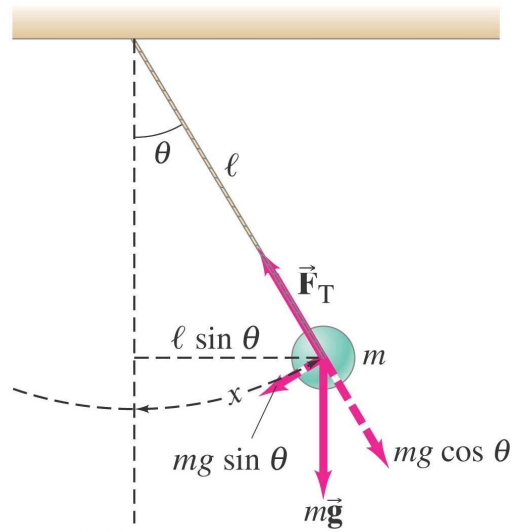
The bottom of the drum is at a depth  $d$  below the surface. Express all answers in terms of the quantities given above, the acceleration due to gravity,  $g$  and the density  $\rho$  of water.

(a) Determine  $d$  in terms of  $M$ ,  $\rho$  and  $A$ .

(b) Suppose an external force of magnitude  $F$  is exerted on the top of the drum, pushing it further down into the water by an extra distance  $x$ . Express  $F$  in terms of  $x$  and the other relevant quantities.

(c) Prove that if the external force is suddenly removed, the drum will execute simple harmonic motion in the vertical direction.

(d) Determine the angular frequency of these vertical oscillations.



9. A pendulum that was originally erected by Foucault at the Pantheon in Paris for the Paris Exhibition in 1851 was restored in 1995. It has a 28.0 kg sphere suspended from a 67.0-m light cable. If the amplitude of the swing is  $x = 5.00$  m (as shown in the above figure), what is the maximum speed of the sphere?