Physics 112 **Problem Set #3** **Winter 2000**

DUE: THURSDAY JANUARY 27, 2000

MIDTERM ALERT: The first midterm exam will be given in class on Tuesday February 1, 2000. This exam will cover material from the first three problem sets. More details next week.

Problems 6 and 7 are taken from Baierlein. *In order to earn total credit for a problem solution, you must show all work involved in obtaining the solution.* Late homework received before Tuesday February 1 will be graded at half credit. No homework sets accepted after solution sets are handed out.

1. Suppose it is known that 1% of the population have a certain kind of cancer. It is also known that a test for this kind of cancer is positive in 99% of the people who have it but is also positive in 2% of the people who do not have it. What is the probability that a person who tests positive has cancer of this type?

2. In this problem, assume for simplicity that a year has 365 days.

(a) What is the probability that two people (selected at random) have different birthdays?

(b) What is the probability that three people (selected at random) have three different birthdays?

(c) Derive an expression for the probability p_n that n people (selected at random) have n different birthdays. Estimate p_n for $n \ll 365$ by computing $\ln p_n$. *HINT:* Make use of the approximation $\ln(1+x) \simeq x$ for $x \ll 1$.

(d) Find the smallest value of n such that $p_n < \frac{1}{2}$.

3. Consider the possible distributions of 2 balls which can be placed in three containers.

(a) Assuming that the balls are distinguishable, enumerate all the possible arrangements of the balls, assuming each ball is placed in one of the containers. (If two balls are placed in one container, assume that the order in which the balls are placed is not pertinent.) Check that the total number of arrangements, and the number of arrangements for each possible distribution of the balls among the containers are given by the general formulas derived in class.

(b) Now assume that the balls are indistinguishable. How many distinct arrangements listed in part (a) remain? Check that this number is correct by comparing to the general formula derived in class.

(c) Assume the balls are indistinguishable and that at most one ball can be placed in a given container. How many distinct arrangements listed in part (a) remain? Check that this number is correct by comparing to the general formula derived in class.

4. Let p be the probability of success and $q = 1 - p$ be the probability of failure. According to the binomial distribution, the probability $f(r)$ of exactly r successes in n trials is given by $f(r) = C(n,r)p^r q^{n-r}$, where $C(n,r) \equiv n!/[r!(n-r)!]$.

(a) Compute the mean of the binomial distribution.

(b) Compute the variance of the binomial distribution.

(c) In class, we proved that for large n, the binomial distribution is well approximated by a continuous Gaussian distribution. Compute the mean and variance of the corresponding Gaussian distribution and compare with the results of parts (a) and (b).

5. Consider the one-dimensional spin system, discussed in class, consisting of N spins each of magnetic moment m_B in a magnetic field B. The spin excess is $2s$. Assuming $|s| \ll N$, it was shown in class that the multiplicity of states was approximately equal to:

$$g(N,s) \simeq g(N,0)e^{-2s^2/N} .$$

(a) Compute the entropy as a function of N and the total energy E. Your answer will also depend on the constant parameters B, m_B and $g(N,0)$.

(b) Evaluate the temperature as a function of N and E.

(c) Find the equilibrium value of the fractional magnetization, defined as $\langle M \rangle/(Nm_B) \equiv 2\langle s \rangle/N$, in terms of the temperature T.

6. Chapter 5, problem 3 (p. 112).

7. Chapter 5, problem 4 (pp. 112–113).

8. *EXTRA CREDIT—have some fun with this.* The following question, called the *Monty Hall Problem* after the television game show host, Monty Hall, was correctly answered by the newspaper columnist Marilyn vos Savant in her weekly column. Her solution stimulated thousands of letters, many from professors of mathematics and statistics (at least so they asserted), that claimed that her answer was incorrect. A subsequent column by vos Savant argued that her solution was indeed correct (and it was!). Here is the problem.

A game show contestant is presented with three closed doors. Behind two of the doors are goats and behind the third door is a brand new car. The contestant chooses a door without opening it. Monty Hall, who knows the location of the car, always goes to one of the other doors and opens it to reveal a goat.[1] The contestant is then given the opportunity of switching to the other unopened door. Is it to the contestant's advantage to switch, to remain with the original choice, or does it make no difference? Specifically, after Monty Hall reveals the location of the first goat, what is the probability that the contestant wins the car if:

(a) the contestant remains with the original choice?

(b) the contestant switches to the other unopened door?

Are you sure? You don't want to embarrass yourself like those math and statistic professors, now do you?

HINT: Here is a variation to think about. Suppose that the game consisted of n doors, with one car and $n-1$ goats, where n is large. You choose one of the doors. Then Monty Hall opens up $n-2$ doors to reveal $n-2$ goats. There are now two doors left: the one you chose and one other that has not been opened. Should you switch? What is the probability of winning the car in cases (a) and (b) now?

[1] If the contestant has actually chosen the door which leads to the car, then you should assume that Monty Hall chooses at random one of the other two doors to open thus revealing the goat.