FINAL EXAM INSTRUCTIONS: You have 3 hours to complete this exam. This is an open book exam. You are permitted to consult the course textbook (Baierlein) and two sheets of handwritten material. No other consultations or collaborations are permitted during the exam. At the end of the exam please hand in all written work including your two sheets of handwritten notes.

PART I: *In questions #1–5, answer each question TRUE or FALSE, and explain your answer briefly. A few sentences (with an illustrative diagram if relevant) should be sufficient. No credit will be given without an accompanying explanation. Each question is worth four points.*

1. Liquid water and water vapor are contained in a cylinder at 100°C and 1 atm pressure. Assume that the system is maintained at fixed temperature in thermodynamic equilibrium. If the piston is pushed down slowly on the system, the pressure of the liquid/gas mixture will increase.

2. Two isolated systems are characterized by different chemical potentials, μ_1 and μ_2. The systems are then placed in thermal and diffusive contact. To reach thermodynamic equilibrium, particles will diffuse from the region of higher chemical potential to the region of lower chemical potential.

3. Consider an ideal quantum gas made up of N particles (where $N \gg 1$). At $T = 0$, the total internal energy of the gas is zero in the case of a Bose gas *and* in the case of a Fermi gas. (If this statement is not correct, how should it be modified so that it is a true statement?)

4. Consider an isolated system at fixed temperature and pressure. Suppose that the system is initially out of thermodynamic equilibrium. Then, equilibrium is reached when the Gibbs free energy attains its minimum.

5. A classical ideal gas violates the third law of thermodynamics.

PART II: *In questions #6–10, Choose the best answer among the choices given. Justify your choice with a short computation or comment. No credit will be given without an accompanying explanation. Each question is worth four points.*

6. Two identical systems, with heat capacity at constant volume that varies as the cube of the temperature, $C_V = bT^3$ (where b is some constant), are thermally isolated. Initially, one system is at temperature 100 K and the other is at temperature 200 K. The systems are then brought into thermal contact and the combined system is allowed to reach thermal equilibrium. The final temperature of the combined system is roughly:

(a) 150 K

(b) 160 K

(c) 165 K

(d) 170 K

(e) Not enough information is given.

7. Deuterium (D) is an isotope of hydrogen (H) with a molecular weight of two atomic mass units. Consider two vessels, one containing H_2 gas and one containing D_2 gas. Each vessel is maintained at the same temperature. The ratio of the average speeds of the molecules in the respective vessels, $\langle v \rangle_{H_2} \,/\, \langle v \rangle_{D_2}$ is:

(a) 1

(b) 2

(c) 1/2

(d) $\sqrt{2}$

(e) $1/\sqrt{2}$

8. Choose the one statement below that *incorrectly* describes the van der Waals equation of state.

(a) It attempts to take into account the finite size of the molecules.

(b) It attempts to take molecular interactions into account.

(c) It represents a state of mechanical equilibrium for all values of P, V, and T that satisfy the equation of state.

(d) It possesses a critical point.

(e) It can be used to model the liquid/vapor phase transition.

9. A substance is prepared such that its solid and liquid phases coexist in thermal equilibrium. It is then found that when the pressure is raised (with the temperature held fixed), the liquid freezes completely. One can conclude that:

(a) Such an occurrence is impossible.

(b) During the freezing process, the specific volume decreases.

(c) During the freezing process, the specific volume increases.

(d) During the freezing process, the specific volume remains constant.

(e) Not enough information is given to determine the behavior of the specific volume during the freezing process.

10. The atom ^{3}He is a spin-1/2 fermion. The density of liquid ^{3}He is 0.081 g cm^{-3}. The Fermi temperature of liquid ^{3}He (in the ideal Fermi gas approximation) is predicted to be about:

(a) 5×10^{-4} K

(b) 0.05 K

(c) 1 K

(d) 5 K

(e) 10 K

PART III: PROBLEMS. *In order to earn total credit for a problem solution, you must show all work involved in obtaining the solution. Each problem is worth twenty points.*

1. A pot of soup boils at 103°C at the bottom of a hill and at 98°C at the top of the hill. If the hill is 0.5 km high, what is the latent heat of vaporization of the soup? (You may assume that the temperature of the air does not vary between the bottom and the top of the hill. If you make any other reasonable assumptions or approximations, be sure to state what they are.)

2. During a portion of the Carnot cycle, a dilute gas of diatomic nitrogen is compressed slowly at constant temperature, $T = 280$ K. The initial volume of the gas is 0.3 m^3 and the final volume of the gas is 0.1 m^3. Assume that the gas consists of 3×10^{24} molecules.

(a) What is the change in multiplicity of the gas? Express the factor by which the multiplicity changes as 10 raised to some power.
(b) What is the change in entropy of the gas?
(c) How much energy was transferred to the environment through heating or cooling. Be sure to specify whether the energy of the environment decreased or increased.
(d) How much work was done on the gas while the gas was being compressed?

3. Stephen Hawking calculated the entropy of a non-rotating uncharged black hole and obtained the expression:

$$S = \frac{kc^3 A}{4G\hbar}\,,$$

where $A \equiv 4\pi R_S^2$ is the surface area of the black hole and $R_S = 2GM/c^2$ is the radius of a black hole of mass M (as predicted in general relativity). The internal energy of a black hole is given by its rest-mass energy, $E = Mc^2$.

(a) Compute the temperature of the black hole. Express your final result in terms of the mass of the black hole and fundamental constants (Boltzmann's constant k, Planck's constant $\hbar \equiv h/2\pi$, the speed of light c and Newton's gravitational constant G). [See the *HINT* following part (b).]

(b) Compute the heat capacity of the black hole. Is it positive or negative? What happens to the black hole temperature as it loses energy? (Prepare to be surprised.)

HINTS for parts (a) and (b): You should find it to your advantage to first substitute for the mass M in all expressions in terms of the energy E.

(c) Hawking's great discovery is that a black hole actually radiates* as if it were a blackbody at temperature T. When black holes radiate, they lose mass. Using the Stefan-Boltzmann law, find a simple differential equation for the rate of change of the mass of a black hole as a function of time due to its energy loss from radiation.

(d) Solve the differential equation obtained in part (c). Show that for a black hole of mass 2×10^{11} kg, the black hole will completely evaporate in a time of $t = 7 \times 10^{17}$ s, which is roughly the age of the universe.

---

*Classically, nothing can escape a black hole. But when quantum mechanical effects are taken into account, Hawking found that black holes do in fact radiate.