Physics 112 **First Midterm Exam** **February 1, 2000**

MIDTERM EXAM INSTRUCTIONS: You have 75 minutes to complete this exam. This is a closed book exam, although you are permitted to consult one sheet of handwritten material. No other consultations or collaborations are permitted during the exam. At the end of the exam please hand in all written work including your one sheet of handwritten notes.

PART I: *In questions #1–8, answer each question TRUE or FALSE, and explain your answer briefly. A few sentences (with an illustrative diagram if relevant) should be sufficient. No credit will be given without an accompanying explanation. Questions #1–8 are worth six points each.*

1. For a monatomic ideal gas, the pressure is given by $P = \frac{2}{3}u$, where u is the energy density ($u \equiv E/V$).

2. Suppose we are given two cylinders, each containing 1 mole of an ideal gas, under identical conditions. We now reduce the pressure in each cylinder by a factor of two. In cylinder 1, the pressure is reduced isothermally, and in cylinder 2, the pressure is reduced adiabatically. Then, the final volumes must satisfy $V_1 > V_2$.

3. In any adiabatic process, the entropy of the system does not change.

4. A gas at temperature T_1 occupies a volume V_1. Suddenly, it expands freely into a vacuum. When thermal equilibrium is reached, the gas occupies a volume $V_2 > V_1$ and is found to be at temperature T_2. If the gas is a classical ideal gas, then $T_2 < T_1$.

5. The second law of thermodynamics states that the entropy of any system can never decrease.

6. Even though $q = TdS$ and $w = PdV$ are true for reversible processes only; nevertheless, the equation $dE = TdS - PdV$ is also true for irreversible processes between equilibrium states.

7. Given two equilibrium states, one can always find an irreversible path in which the change of entropy is *larger* than the entropy change along a reversible path connecting the same two states.

8. A Carnot engine whose thermal efficiency is very high is particularly well suited as a refrigerator, if it is run in the reversed direction.

PART II: PROBLEMS. *In order to earn total credit for a problem solution, you must show all work involved in obtaining the solution. The point value of each problem is indicated in the square brackets below.*

1. [27] A vertical cylinder contains 500 moles of a monatomic ideal gas and is closed off by a piston of mass 50 kg and a cross-sectional area of 100 cm^2. The whole system is thermally insulated during the entire process described below. Initially, the piston is clamped in position so that the gas occupies a volume of 1 m^3 and is at a temperature of 300 K. The piston is then released, and eventually comes to rest in a final equilibrium position corresponding to some larger volume V of the gas. Neglect any frictional forces which might prevent the piston from sliding freely within the cylinder. Compute:

(a) the number of gas molecules in the cylinder;

(b) the initial pressure of the gas (in units of atm); and

(c) the final pressure of the gas (in units of atm).

(d) Obtain an expression for the work done by the expanding gas in terms of change in volume of the system.

Using the above results, apply the first law of thermodynamics and compute:

(e) the final volume of the gas; and

(f) the final temperature of the gas.

[*HINT:* The final equilibrium position of the piston is reached when the weight of the piston is exactly balanced by the pressure of the gas. Note: the process described above is *not* reversible.]

2. [25] Consider a collection of N identical non-interacting quantum oscillators. The energy levels of the jth oscillator is given by $E_j = n_j \hbar\omega$, where n_j is a non-negative integer, $\hbar$ is Planck's constant and ω is the angular frequency of each oscillator.[1] The total energy of the system is given by

$$E = \sum_{j=1}^{N} E_j = \hbar\omega \sum_{j=1}^{N} n_j \,. \tag{1}$$

Let us define the integer n by $E = \hbar\omega n$. Then, using eq. (1), it follows that

$$n = \sum_{j=1}^{N} n_j \,. \tag{2}$$

Given a system of N oscillators with total energy E, the multiplicity is given by the number of ways of choosing N non-negative integers $n_1, n_2, \ldots, n_N$ whose sum is equal to n [eq. (2)].

(a) Show that the multiplicity as a function of n and N is given by:

$$g(N, n) = \frac{(N + n - 1)!}{n!(N - 1)!} \,.$$

HINT: Note the relation between this problem and the problem of counting the number of ways of distributing n indistinguishable balls in N boxes.

(b) Compute the entropy of the system of N oscillators with total energy E. Express your result in terms of N and E, under the assumption that $N \gg 1$ and $E \gg \hbar\omega$. (Simplify your final expression by approximating $N - 1 \simeq N$.)

(c) From the result of part (b), compute the thermodynamic temperature, T.

(d) Let $\epsilon \equiv E/N$ be the average energy per oscillator at temperature T. Use the result of part (c) to derive an expression for ϵ as a function of the temperature T.

(e) Evaluate ϵ/kT in the limit of large temperature (*i.e.*, $kT \gg \hbar\omega$).

[1] I am neglecting the zero-point energy of the quantum oscillators. It is correct to do so if one measures all energies relative to the ground-state energy. By dropping the zero-point energy, one effectively defines the ground state energy to be equal to zero.

1. For your convenience, I list some of the fundamental constants and conversions below:

$$
\begin{aligned}
k &= 1.381 \times 10^{-23} \text{ J K}^{-1} \\
N_A &= 6.022 \times 10^{23} \text{ mole}^{-1} \\
g &= 9.8 \text{ m s}^{-2}
\end{aligned}
$$

where k is Boltzmann's constant, N_A is Avagadro's number and g is the acceleration due to gravity.

One atmosphere pressure is related to the corresponding MKS unit by the following relation:

$$1 \text{ atm} = 1.013 \times 10^5 \text{ N m}^{-2} .$$

2. Stirling's approximation is given by:

$$\ln N! = (N + \tfrac{1}{2}) \ln N - N + \tfrac{1}{2} \ln 2\pi + \mathcal{O}\left(\frac{1}{N}\right) .$$

Often, it is sufficient to use the slightly less accurate result:

$$\ln N! \simeq N \ln N - N .$$

In problem 2, the latter result is sufficiently accurate.