DUE: THURSDAY October 24, 2019

To receive full credit, you must exhibit the intermediate steps that lead you to your final results.

1. Boas, p. 67, problem 2.10–20.

2. Boas, p. 67, problem 2.10–28.

3. Boas, p. 69, problem 2.11–18.

4. Boas, p. 71, problem 2.12–27.

5. Boas, p. 74, problem 2.14–9. Note that the 90° rotation about the origin in this problem should be taken in the *counterclockwise* direction.

6. Boas, p. 74, problem 2.14–24.

7. Boas, p. 76, problem 2.15–6.

8. Boas, p. 76, problem 2.15–7.

9. Boas, p. 77, problem 2.16–1.

10. Boas, p. 79, problem 2.16–11.

11. Evaluate $\operatorname{Arg}(1 + e^{i\theta})$ as a function of θ, where $-\pi < \theta < \pi$, using two different methods:

(a) Write the complex number $z = 1 + e^{i\theta}$ in polar form. Identify the magnitude $|z|$, and then determine $\operatorname{Arg} z$.

(b) Use a graphical technique by drawing a triangle in the complex plane, whose three vertices correspond to the complex numbers 0, 1 and $1 + e^{i\theta}$. Determine the angles of the triangle. Using this picture, what is the value of $\operatorname{Arg}(1 + e^{i\theta})$?

(c) Identify the curve in the complex plane described by the equation $z = 1 + e^{i\theta}$ as θ varies over its allowed range, $-\pi < \theta < \pi$.

12. The series for the principal value of the complex-valued logarithm,

$$\mathrm{Ln}(1-z) = -\sum_{n=1}^{\infty} \frac{z^n}{n}\,, \tag{1}$$

converges for all $|z| \leq 1$, $z \neq 1$. In particular, consider the conditionally convergent series,

$$S \equiv \sum_{n=1}^{\infty} \frac{e^{in\theta}}{n}\,, \qquad \text{where } 0 < \theta < 2\pi\,. \tag{2}$$

(a) By taking the real part of eq. (2), evaluate

$$\sum_{n=1}^{\infty} \frac{\cos n\theta}{n}\,, \qquad \text{where } 0 < \theta < 2\pi\,,$$

as a function of θ. Check that your answer has the right limit for $\theta = \pi$.

(b) By taking the imaginary part of eq. (2), prove that

$$\sum_{n=1}^{\infty} \frac{\sin n\theta}{n} = \tfrac{1}{2}(\pi - \theta)\,, \qquad \text{where } 0 < \theta < 2\pi\,.$$

<u>HINT:</u> The simplest method for solving this problem is to first sum the series given in eq. (2) [using the result of eq. (1)]. Then, simplify the resulting expression for the complex-valued logarithm by making use of one of the two methods employed in the previous problem. Finally, take the real and imaginary parts of the final result to solve parts (a) and (b) above.