DUE: THURSDAY, OCTOBER 22, 2009

MIDTERM ALERT: On Thursday, October 29, the midterm exam will take place during the usual class hours in ISB 231. The exam will cover material from the first nine lectures of this course. During the exam, you may refer to the textbook, the class handouts, or your own personal notes. Collaboration with your neighbor, of course, is strictly forbidden.

1. Liboff, problem 9.32 on page 395.

2. Liboff, problem 9.46 on page 400.

3. Liboff, problem 11.61 on page 541.

4. The current density $\vec{\boldsymbol{J}}$ (in the absence of an external electromagnetic field) was introduced by Liboff in eq. (7.107) on p. 219. The corresponding number density is given by $\rho = |\psi(\vec{\boldsymbol{r}})|^2$. Both quantities are related through the continuity equation [eq. (7.97) of Liboff on p. 217]. It is convenient to redefine these quantities by multiplying them by the charge e of the particle. Then, the corresponding number density is reinterpreted as the charge density and the corresponding current density is reinterpreted as the electromagnetic current density.

(a) Show that the electromagnetic current density for a particle with charge e in an external electromagnetic field (in cgs units) is given by:

$$\vec{\boldsymbol{J}} = \frac{e\hbar}{2im}\left(\psi^*\vec{\boldsymbol{\nabla}}\psi - \psi\vec{\boldsymbol{\nabla}}\psi^*\right) - \frac{e^2}{mc}\vec{\boldsymbol{A}}\psi^*\psi = \frac{e}{m}\mathrm{Re}\left[\psi^*\left(\vec{\boldsymbol{p}} - \frac{e\vec{\boldsymbol{A}}}{c}\right)\psi\right] ,$$

where $\vec{\boldsymbol{A}}$ is the electromagnetic vector potential and $\vec{\boldsymbol{p}} = -i\hbar\vec{\boldsymbol{\nabla}}$.

HINT: Start from the Schrodinger equation for a charged particle in an external electromagnetic field (in the Coulomb gauge), and repeat the derivation of eq. (7.107) of Liboff.

(b) Show that the continuity equation remains valid. Is the definition of ρ modified as compared to the case of zero external electromagnetic field?

5. Let $X(\vec{\boldsymbol{r}}, t)$ be an arbitrary function of $\vec{\boldsymbol{r}}$ and t, and let $\psi(\vec{\boldsymbol{r}}, t)$ be the solution to the Schrodinger equation for a charged particle (with charge e) in an external electromagnetic field.

(a) Consider a new wave function $\psi_1(\vec{\boldsymbol{r}}, t)$ defined by

$$\psi_1(\vec{\boldsymbol{r}}, t) = \left[\exp\left(\frac{ieX(\vec{\boldsymbol{r}}, t)}{\hbar c}\right)\right] \psi(\vec{\boldsymbol{r}}, t) \,.$$

Find the Schrodinger equation that is satisfied by $\psi_1(\vec{\boldsymbol{r}}, t)$.

(b) Show that $\psi_1(\vec{\boldsymbol{r}}, t)$ represents a solution to the Schrodinger equation for a particle in a new electromagnetic scalar and vector potential that are related to the original scalar and vector potential by a gauge transformation.