

35. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, e.g., for $-8/15$ read $-\sqrt{8/15}$.

Notation:

J	J	...
M	M	...
m_1	m_2	
m_1	m_2	Coefficients
$\vdots$	$\vdots$	
$\vdots$	$\vdots$	

$1/2 \times 1/2$

1	0
+1/2 +1/2	1 0 0
+1/2 -1/2	1/2 1/2 1
-1/2 +1/2	1/2 -1/2 -1
-1/2 -1/2	1

 $Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos \theta$
 $2 \times 1/2$

5/2	3/2
+5/2	1
+2 +1/2	1/5 4/5
+1 +1/2	4/5 -1/5
	5/2 3/2
	1/2 +1/2

 $Y_1^1 = -\sqrt{\frac{3}{8\pi}} \sin \theta e^{i\phi}$
 $Y_2^0 = \sqrt{\frac{5}{4\pi}} \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2} \right)$
 $3/2 \times 1/2$

2	1
+2	1
+3/2 +1/2	1/4 3/4
+1/2 +1/2	3/4 -1/4
	2 1
	0 0

 $Y_2^1 = -\sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{i\phi}$
 $Y_2^2 = \frac{1}{4} \sqrt{\frac{15}{2\pi}} \sin^2 \theta e^{2i\phi}$

$1 \times 1/2$

3/2	1/2
+3/2	1
+1 +1/2	1/3 2/3
0 +1/2	2/3 -1/3
	3/2 1/2
	-1/2 -1/2

 2×1

3	2
+3	1
+2 0	1/3 2/3
+1 1	2/3 -1/3
	3 2 1
	+1 +1 +1

 $3/2 \times 1$

5/2	3/2
+5/2	1
+3/2 +1	2/5 3/5
+1/2 +1	3/5 -2/5
	5/2 3/2
	1/2 +1/2

 1×1

2	1
+2	1
+1 0	1/2 1/2
0 +1	1/2 -1/2
	2 1 0
	0 0 0

 $2 \times 1/2$

3/2	1/2
+3/2	1
+1 0	1/2 1/2
0 +1	1/2 -1/2
	2 1 0
	0 0 0

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 $2 \times 3/2$

7/2	5/2
+7/2	1
+2 +1/2	3/7 4/7
+1 +3/2	4/7 -3/7
	7/2 5/2
	3/2 +3/2

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	1/2 1/2
+1/2 +3/2	1/2 -1/2
	3 2 1
	+1 +1 +1

 2×2

4	3
+4	1
+2 +1	1/2 1/2
+1 +2	1/2 -1/2
	4 3 2
	+2 +2 +2

 $3/2 \times 3/2$

3	2
+3	1
+3/2 +1/2	