Corrections to Relativity, Gravitation and Cosmology 2e
Cheng (10-1-2015)

- p.46, Sidenote 11: Insert an equation reference "(12.17)" in the 3rd sentence so the Sidenote will read

  [11] A more formal...... Such a transformation, cf.(12.17), must satisfy the "generalized orthogonality condition",....

- p.49, above Eq.(3.39): Insert a new Sidenote 15*a* at the end of the text prior to the displayed (3.39):

  [15a] In terms of the 4-velocity components, we have

  $$p^i = mdr^i/d\tau \quad \text{and} \quad E = mc^2 dt/d\tau$$

- p.84, above Eq.(5.10): Insert a new Sidenote 3a at the end of the text prior to the displayed (5.10):

  [3a] Recall that a matrix multiplication involves the summation of a pair of row and column indices and that the indices a and b on g_{ab} are respectively the row and column indices.

- p.131, the line above Eq.(7.47): Insert a new Sidenote 14*a* at the word "energy[14a]"

  [14a] In the SR limit of $r^* = 0$, it reduces to the energy $E = mc^2 dt/d\tau$ as shown in Sidenote 15a on p.49.

- p.149, the 4th line from the bottom: Insert a new Sidenote 5*a* at the word "line" in the text "... $r = 0$ line[5a] so that..."

  [5a] In the $r < r^*$ region the lightlike geodesics of (8.14) $cd\bar{t} = -dr$ and (8.22) $cd\bar{t} = dr\,(r + r^*)\,/\,(r - r^*)$ are both incoming as r decreases with an increasing $\bar{t}$.

- p.159, the 1st line of the second paragraph: Insert a new Sidenote 12*a* at the end of the line as in ".Oppenheimer and Snyder performed a GR study[12a] and made most...."

  [12a] Their research on gravitational collapse showed analytically that a cold Fermi gas quickly collapses from a smooth initial distribution to form a black hole with the properties discussed above.

- p.174, the 3rd line below Eq.(8.80): Insert a new Sidenote 25 at the end of the sentence "...area is ever-increasing[25]."

  [25] A spherical mass dM falling into a BH, the initial area $(A^* + dA^*) \propto (M^2 + dM^2)$ is clearly less than the final state BH area $\propto (M + dM)^2$.

- p.197, Eq.(9.37): Insert a set of missing parentheses [...]. It should appear as "$dl^2 = R_0^2 a^2 (t) \left[d\chi^2 + k^{-1} \left(\sin^2 \sqrt{k}\chi \right) d\Omega^2 \right]$".

- p.204, Problem 9.11(b): Change in the text "Problem 9.10" to "Problem 9.9".

- p.216, Eq.(10.35): Replace h by $\hbar$ ("h bar") so as to have

$$n_b = \frac{4}{3} n_f = 2.404 \frac{g}{2\pi^2} \left(\frac{k_B T}{\hbar c} \right)^3$$

- p.217, the line above Eq.(10.43): Insert a new Sidenote $8a$ at the end of the phrase "leading to[8a]"

  [8a] The effective spin degrees of freedom $g^* = 2\,(photon) + \frac{7}{8} \times 4(e^+, e^-) + \frac{7}{8} \times 6(\nu, \bar{\nu}) = 10\frac{3}{4}$.

- p.221, Eq.(10.53): Change the value from "0.7 MeV"to "0.1 MeV"so the displayed equation reads as $k_B T_{bbn} \backsimeq 0.1$ MeV,"

- p.226, Eq.(10.65): Replace h by $\hbar$ so that the equation reads as

$$n_{\gamma,0} = \frac{2.4}{\pi^2} \left(\frac{k_B T_{\gamma,0}}{\hbar c} \right)^3 \backsimeq 411/\text{cm}^3.$$

- p.226 - 227, the paragraph below Eq.(10.66): Modify the entire paragraph except the first sentence. Namely replace the sentences

  "This also explains why the thermal energy at photon decoupling is as low as 0.26 eV. .... Thus, even though the average photon energy was only 0.70 eV, there remained up to that time a sufficient number of high energy photons at the tail end of the distribution to hold off the transition to a new equilibrium phase."

  by the sentences

  "Considering that the average photon energy at decoupling is $2.7 \times 0.26 \simeq 0.70$ eV, why was the ionization not shut off until the thermal energy fell so much below the hydrogen ionization energy of 13.6 eV? This just reflects the fact that there are so many photons for every baryon that even though the average photon energy was only 0.70 eV, there remained up to that time a sufficient number of high energy photons at the tail end of the distribution to hold off the transition to a new equilibrium phase.".

- p.227, Eq.(10.67): Replace "$k_B T_{\gamma,0}$" by "0.70 $eV/(1100)$", and replace "8.8" by "3.3" so that the displayed equation reads as

$$\begin{aligned}\frac{\Omega_{\mathrm{M},0}}{\Omega_{\mathrm{R},0}} &= \frac{\Omega_{\mathrm{M},0}}{\Omega_{\mathrm{B},0}} \frac{\Omega_{\mathrm{B},0}}{1.68 \times \Omega_{\gamma,0}} \\ &\simeq \frac{0.25}{0.04} \frac{n_{\mathrm{B}}}{n_\gamma} \frac{m_{\mathrm{N}} c^2 \times 1100}{1.68 \times 0.70} \simeq 3.3 \times 10^3, \end{aligned} \tag{10.67}$$

- p.227, 3rd line below Eq.(10.67): Replace "$k_B T_{\gamma,0}$" by "0.70 $eV/(1100)$ as the average value was 0.70 eV at decoupling ($z = 1100$)" so that the sentences reads

  " .... and photon energy by 0.70 $eV/(1100)$ as the average value was 0.70 eV at decoupling ($z = 1,100$). The baryon photon ratio ...."

- p.227-228, Eq.(10.68) and the paragraph above and below Eq.(10.69): Replace in Eq.(10.68) the factor "1.1×10^4" by "3300", replace on the 1st line below Eq.(10.68) the factor "8800" by "3300", on the 2nd line replace the word "eight" by the word "three", on the 3rd line replace the factor "8" by "3", and in Eq.(10.69) replace the numbers "8" by "3" and "2" by "1", respectively, as well as on 2nd line from the top of p.228, replace the number "16,000" by "70,000" so that the new Eq.(10.68), Eq.(10.69) and the paragraphs that follow will read as "

$$1 = \frac{\rho_{\mathrm{R}}}{\rho_{\mathrm{M}}} = \frac{\rho_{\mathrm{R},0}}{\rho_{\mathrm{M},0}} [a(t_{\mathrm{RM}})]^{-1} = \frac{\Omega_{\mathrm{R},0}}{\Omega_{\mathrm{M},0}} (1 + z_{\mathrm{RM}}) \simeq \frac{1 + z_{\mathrm{RM}}}{3300}. \tag{10.68}$$

  Hence the redshift for radiation–matter equality is $z_{\mathrm{RM}} \simeq 3,300$, which is three times larger than the photon decoupling time with $z_\gamma \simeq 1,100$. This also yields scale factor and temperature ratios of $[a(t_\gamma)/a(t_{\mathrm{RM}})] = [T_{\mathrm{RM}}/T_\gamma] \simeq 3$, or a radiation thermal energy

$$k_B T_{\mathrm{RM}} = 3\ k_B T_\gamma = O(1\ \mathrm{eV}). \tag{10.69}$$

  Knowing this temperature ratio we can find the "radiation–matter equality time" $t_{\mathrm{RM}} \simeq 70,000$ yrs (Problem 10.9) from the photon decoupling time $t_\gamma \simeq 360,000$ yrs. From that time t_{RM} on, gravity (less opposed by significant radiation pressure) began to grow, from the tiny lumpiness in matter distribution, the rich cosmic structures we see today."

- p.229, 3rd line below (10.75): Insert a new Sidenote 20a at the at the word "densities" as in "... and neutrino number densities^{20a} as first stated in Section 10.3.2." The new sidenote reads as

  ^{20a} From the neutrinos' temperature, one can fix their number density via (10.35). Because neutrinos are fermions and photons are bosons, we have

$$\frac{n_\nu}{n_\gamma} = \frac{3}{4} \left(\frac{T_\nu}{T_\gamma}\right)^3 \frac{g_\nu}{g_\gamma}$$

As there are two photon states and six neutrino states (left-handed neutrinos and right-handed antineutrinos for each of the three lepton flavors). Plugging in these multiplicities and the temperature ratio from (10.75) yields

$$\frac{n_\nu}{n_\gamma} = \frac{9}{11}.$$

- p.236, Problem 10.6, the 1st line on top: Replace the phrase "the average photon energy was $\bar{u} \simeq 0.26\ eV$" by "and the average thermal energy was $k_{\rm B}T_\gamma \simeq 0.26\ eV$."

- p.236, Problem 10.12, the displayed equation: Insert vertical bars around the factor "$\Omega_0 - 1$" so that it will read as

$$R_0 = \frac{c}{H_0\sqrt{|\Omega_0 - 1|}}.$$

- p.236, Replace the entire Problem 10.13 by the new wordings:

"**10.13** Cosmological limit of neutrino mass (9.27) and (9.28) inform us that the dark matter density parameter being $\Omega_{DM} = \Omega_M - \Omega_B \simeq 0.21$, if we assume that this dark matter is composed entirely of light neutrinos and antineutrinos, what limit can be obtained for the average mass of neutrinos (averaged over three flavors ν_e, ν_μ, and ν_τ)?"

- p.263, side note 28: Insert a pair of missing square brackets in the equation's 1st line and replace the equation's 2nd line:

$$\begin{aligned} t(a) &= t_{\rm H}\int_0^a [\Omega_{\rm M,0}/a' + \Omega_\Lambda a'^2]^{-1/2} da' \\ &= \frac{2t_{\rm H}}{3\sqrt{\Omega_\Lambda}} \sin^{-1}\left[\frac{a^{3/2}}{(\Omega_{\rm M,0}/\Omega_\Lambda)^{1/2}}\right]. \end{aligned}$$

- p.263, 3rd line below Eq.(11.48): Change the number 7 to 8.6 so that part of the sentence reads as "... of $t_{\rm tr} = t(a = 0.56) \simeq 8.6\,\text{Gyr}$—in cosmic..."

- p.275, Problem 11.3, the displayed equation: Delete a factor of c and move the factor H_0 out of the integral sign, so that the equation reads as

$$d_{\rm L}(z) = \frac{c\,(1+z)}{H_0}\int_0^z \frac{dz'}{\left[\Omega_{\rm M,0}\,(1+z')^3 + \Omega_\Lambda\right]^{1/2}}.$$

- p.275, Problem 11.5: Replace the problem by the new wordings:

11.5 Estimate of matter and dark energy equality time Closely related to the deceleration/acceleration ("inflection") transition time t_{tr} is the epoch $t_{\mathrm{M}\Lambda}$ when the matter and dark energy components are equal $\Omega_{\mathrm{M}}(t_{\mathrm{M}\Lambda}) = \Omega_{\Lambda}(t_{\mathrm{M}\Lambda})$. Show that the estimated matter and dark energy equality time $t_{\mathrm{M}\Lambda}$ is comparable to, and as expected somewhat greater than, $t_{\mathrm{tr}} \simeq 8.6$ Gyr of (11.48)."

- p.281, the paragraph below (12.6): After the 1st sentence revise the paragraph including the interchange of the words "contravariant" and "covariant" so that the paragraph reads as

  "Repeated indices are summed in the expansions of the vector **A**. However, as a shorthand, it is common practice to refer A_μ or A^ν as the vector **A** itself, even though A_μ and A^ν are technically the covariant and contravariant components, respectively, of the vector **A**. In this convention, A_μ is called a covariant vector and A^ν a contravariant vector."

- p.283, Sidenote 7: Add a sentence at the end of the sidenote, "Cf. also Sidenote 9a on p.287."

- p.287, above Eq.(12.40): Insert a new Sidenote 9*a* at the end of the text prior to the displayed (12.40):

  [9a] Recall comments made in Sidenote 3*a* on p.84.

- p.288, at the end of Box 12.1: Reverse the order of superscript indices ($\mu \leftrightarrow \nu$) on $F^{\mu\nu}$ in both Eqs.(12.41) and (12.42) so they read as

$$F^{\nu\mu}F_{\mu\nu} = 2\left(\vec{E}^2 - \vec{B}^2\right) \tag{12.41}$$

  and

$$F^{\nu\mu}\tilde{F}_{\mu\nu} = 4\left(\vec{E}\cdot\vec{B}\right). \tag{12.42}$$

- p.311, Eq.(13.57): Insert a minus sign on the right hand side

$$dA^\mu = -R^\mu{}_{\nu\lambda\rho}A^\nu a^\lambda b^\rho. \tag{13.57}$$

  and add a new Sidenote 8*a* at the end of the text above the displayed equation (13.57):

  [8a] The minus sign is required so as to be compatible with the curvature definition given in (13.58), if the direction of the parallel transport loop is in accord with the area direction (13.56), *i.e.*, given by the right-hand rule around $\boldsymbol{\sigma}$ in the 2D case (counterclockwise in Fig. 13.5).

- p.312, Eq.(13.65): Interchange the order of subscript indices ($\alpha \leftrightarrow \beta$) in the last two terms on the right hand side of the displayed equation so it will read as

$$\begin{aligned} dA^{\mu}_{PQ'P'} &= -\Gamma^{\mu}_{\nu\alpha}A^{\nu}b^{\alpha} - \Gamma^{\mu}_{\nu\beta}A^{\nu}a^{\beta} + A^{\nu}\Gamma^{\mu}_{\nu\beta}\Gamma^{\beta}_{\rho\sigma}a^{\rho}b^{\sigma} \\ &\quad -\partial_{\beta}\Gamma^{\mu}_{\lambda\alpha}A^{\lambda}a^{\alpha}b^{\beta} + \Gamma^{\mu}_{\nu\alpha}\Gamma^{\nu}_{\lambda\beta}A^{\lambda}a^{\alpha}b^{\beta}. \end{aligned} \tag{13.65}$$

- p.312, Eq.(13.66): Interchange the order of two terms on the 1st line and insert a minus sign on the 2nd line so that the equation will read as

$$\begin{aligned} dA^{\mu} &= dA^{\mu}_{PQP'} - dA^{\mu}_{PQ'P'} \qquad (13.66) \\ &= -\left[\partial_{\alpha}\Gamma^{\mu}_{\lambda\beta} - \partial_{\beta}\Gamma^{\mu}_{\lambda\alpha} + \Gamma^{\mu}_{\nu\alpha}\Gamma^{\nu}_{\lambda\beta} - \Gamma^{\mu}_{\nu\beta}\Gamma^{\nu}_{\lambda\alpha}\right]A^{\lambda}a^{\alpha}b^{\beta} \end{aligned}$$

- p.313, the line above (13.67): Insert a new Sidenote 9*a* at the end of the text prior to the displayed (13.67):

  [9a] The substitution rule (14.1) $d_x \to D_x$ will help us to see how the expression of curvature by the commutator of covariant derivatives is related to that by transport a vector around an infinitesimal closed path The displacement of a vector A by dx in a curved space may be written schematically as $A(x+dx) = A + dxD_xA = (1+dxD_x)A$ so that the transport of this vector around a closed parallelogram spanned by dx and dy can then be expressed as $(1+dxD_x)(1+dyDy)A - (1+dyD_y)(1+dxD_x)A = dxdy[D_x, D_y]A$.

- p.317, Problem 13.11: Remove 2 minus signs so that Part (a) will have $K = R_{1212}/g$ and Part (b) have $R = 2K$.

- p.325, Eq.(14.32): Remove the minus sign from the first displayed equation so it reads as

$$\Gamma^0_{00} = \frac{\dot{\nu}}{2}$$

- p.326, the 3rd displayed equation from the bottom of the page: Change the minus to a plus sign for the middle term so it reads as

$$-e^{\nu}\left[\frac{d^2x^0}{d\sigma^2} + \nu'\frac{dr}{d\sigma}\frac{dx^0}{d\sigma} + \frac{\dot{\nu}}{2}\left(\frac{dx^0}{d\sigma}\right)^2 + \frac{\dot{\rho}}{2}e^{\rho-\nu}\left(\frac{dr}{d\sigma}\right)^2\right] = 0$$

  Drop the minus sign on the RHS of the middle of Eq.(14.37) so it reads as

$$\Gamma^0_{00} = \frac{\dot{\nu}}{2},$$

- p.335, Problem 14.6, Eq.(14.76): Insert the power 2 so that it reads $\Omega^2 = \frac{G_\mathrm{N} M}{R^3} = \frac{r^*}{2R^3}$.

- p.340-341, Eq.(15.20) and Table 15.1: Insert a minus sign so as to have $\Box\bar{h}_{\mu\nu} = -\frac{16\pi G_\mathrm{N}}{c^4} T^{(0)}_{\mu\nu}$ in (15.20) and in the lower right hand side equation of Table 15.1.

- p.343, Eq.(15.30): Change the + sign into a subscript after the letter h so that it reads as "$= \left[1 + \frac{1}{2}h_+ \sin\omega\,(t - z/c)\right]\xi$."

- p.344, 1st sentence after Eq.(15.31): Change the sentence from "Thus, the separation along the x direction is elongated while along the y direction compressed." to

  "Thus, the separation between two test masses is determined to be an elongation in x direction and a compression in the y direction."

- p.344, 3rd line in the 2nd paragraph "The effect of a wave ...":

  Change the $\times$ sign into a subscript after the letter h so that it reads as "... $ds = [1 \pm \frac{1}{2}h_\times \sin\omega(t - z/c)]\xi$."

- p.349, the line above (15.44) as well as Eqs.(15.44) and (15.45) themselves: Insert the clause ", after dropping higher order terms in $\tilde{h}_+$," just ahead of the last word "are" on the line above the equations, and drop the 2nd terms in the parenthesis so that these two equations with the revised line above them will read as

  "... elements, after dropping higher order terms in $\tilde{h}_+$, are

  $$\Gamma^1_{10} = \Gamma^1_{01} = \Gamma^0_{11} = \frac{1}{2}\partial_0\tilde{h}_+ \tag{15.44}$$

  and, similarly,

  $$\Gamma^1_{13} = \Gamma^1_{31} = -\Gamma^3_{11} = -\frac{1}{2}\partial_0\tilde{h}_+ \tag{15.45}$$

- p.382, Problem 6.2: In 2nd to the last line insert a "slash" in front of g_{00} in the inline equation so it reads as "$\gamma_{ij} = g_{ij} - g_{0i}g_{0j}/g_{00}$".

- p.385, Problem 7.3: the line above the displayed equation

  $$(u_0'' + u_0) + \epsilon\left(u_1'' + u_1 - u_0^2\right) + ... = 0$$

  replace the factor $O\left(r^*\right)$ by $3r^*/2$ so the last inline equation reads as "with $\epsilon = 3r^*/2$,'

- p.385, Problem 7.3: the line above the displayed equation

$$\frac{1}{r} = \frac{\sin\phi}{r_{\rm min}} + \frac{3+\cos 2\phi}{4}\frac{r^*}{r_{\rm min}^2}.$$

  replace the sentence "Putting the zeroth and first order terms together we get" by the new sentence "In this way $u = u_0 + \epsilon u_1$ with $u_1 = (3+\cos 2\phi)\,/\left(6r_{\rm min}^2\right)$:"

- p.395, Problem 10.7, 5th line from the top: Delete the word "negative" and the parentheses so that it reads as "... parameter is $w = -1, ...$".

- p.395, Problem 10.9: in the displayed equation, move the factors of "t_γ" ahead of the brackets, and replace the number "$8^{3/2}$" by "$3^{3/2}$", and "16 000" by "70 000" so the displayed equation becomes

$$t_{\rm RM} = t_\gamma\left[\frac{a(t_{\rm RM})}{a(t_\gamma)}\right]^{3/2} = t_\gamma\left[\frac{1+z_{\rm RM}}{1+z_\gamma}\right]^{-3/2} \simeq \frac{t_\gamma}{3^{3/2}} \simeq 70,000 \text{ years.}$$

- p.395, Problem 10.13, Replace the entire solution as

10.13 Cosmological limit of neutrino mass If we assume that dark matter is made up of neutrinos $\rho_{\rm DM} = n_\nu \bar{m}$, where $\bar{m}$ is the average neutrino mass. The neutrino number density n_ν includes the three flavors ν_e, ν_μ, ν_τ,and their antiparticles. We now relate n_ν to photon densities $n_\gamma$by way of (10.35). First we note that the polarization degrees of a photon $g_\gamma = 2$ and that each light neutrino has only one helicity state, for three flavors of neutrinos and antineutrinos, a total light neutrino degrees of freedom are $g_\nu = 6$ Furthermore neutrino being a fermion and photon a boson, there is a factor of 4/3 difference in their density ratio as shown in (10.35): $n_\nu = \frac{6}{2}\times\frac{3}{4}\left(\frac{T_\nu}{T_\gamma}\right)^3 n_\gamma$. From the neutrino and photon temperature ratio of $T_\gamma = 1.4T_\nu$ obtained in (10.75) and photon number density of $n_\gamma \simeq 410$ cm^{-3} as derived in (10.65), we then have $n_\nu = \frac{9}{4}\left(\frac{1}{1.4}\right)^3 \times 410 \simeq 330$ cm^{-3}. If we identify the neutrino density parameter to the dark matter density $n_\nu \bar{m}c^2/\left(\rho_{\rm c}c^2\right) = \Omega_{\rm DM} \simeq 0.21$, we then obtain the constraint on the average neutrino mass $\bar{m}c^2 \simeq 0.21 \times 5,500/330 \simeq 3.5$ eV,where we have used the critical energy density $\rho_{\rm c}c^2$ value as given in (9.17). This $\bar{m}c^2$ value is for the total neutrino and antineutrino masses for all flavors. Thus, per neutrino of a given flavor, we have the average value of $\left(\bar{m}_\nu c^2\right)_1 \simeq 0.6$ eV

- p.396, Problem 11.4: Insert the three missing square brackets in all three lines of the displayed equations, replace the clause just before the 2nd equation "we calculate in a way similar to that shown in sidenote 28," by "can be calculated", and add a paragraph at the end so as the entire revised solution 11.4 reads as

11.4 Negative Λ and the "big crunch" For the $\Omega_0 = 1$ flat universe with matter and dark energy, we have the Friedmann (11.38) $H(a) = H_0[\Omega_{M,0}a^{-3} + \Omega_\Lambda]^{1/2}$. At $a = a_{\max}$ the universe stops expanding and $H(a_{\max}) = 0$, thus $a_{\max} = (-\Omega_{M,0}/\Omega_\Lambda)^{1/3}$. The cosmic time for the big crunch being twice the time for the universe to go from $a_{\max}$ to $a = 0$, can be calculated

$$2t_H \int_0^{a_{\max}} \frac{da}{[\Omega_{M,0}a^{-1} + \Omega_\Lambda a^2]^{1/2}} = \frac{4t_H}{3\sqrt{-\Omega_\Lambda}} \int_0^{a_{\max}^{3/2}} \frac{dx}{[a_{\max}^3 - x^2]^{1/2}}$$

$$= \frac{4t_H}{3\sqrt{-\Omega_\Lambda}} \left[\sin^{-1}\left(\frac{x}{a_{\max}^{3/2}}\right)\right]_0^{a_{\max}^{3/2}} = \frac{2\pi}{3\sqrt{-\Omega_\Lambda}} t_H = t_*.$$

One could just as easily arrive at this result by directly plugging in $a^3 = a_{\max}^3 = -\Omega_{M,0}/\Omega_\Lambda$ into the relation displayed in sidenote 28 on p.263, and make use of the math relation of $\ln e^{i\theta} = i\theta$ so that $\ln\sqrt{-1} = i\pi/2$.

- p.396, Problem 11.5: Replace the solution by

11.5 Estimate of matter and dark energy equality time We define the matter and dark energy equality time $t_{M\Lambda}$ as $\rho_M(t_{M\Lambda}) = \rho_\Lambda(t_{M\Lambda})$.Using the scaling properties of these densities we have $\rho_{M,0}/a_{M\Lambda}^3 = \rho_{\Lambda,0}$ or $a_{M\Lambda}^3 = \Omega_{M,0}/\Omega_\Lambda$, which differs from the deceleration-acceleration transition scale factor result [cf. (11.47) $a_{tr}^3 = \Omega_{M,0}/2\Omega_\Lambda$ by a factor of $2^{1/3} \approx 1.25$ to yield $a_{M\Lambda} = 0.7$ (and a redshift of $z_{M\Lambda} = 0.42$)] . Plugging in $a = a_{M\Lambda} = (\Omega_{M,0}/\Omega_\Lambda)^{1/3}$ into the formula given in Sidenote 28, we obtain the corresponding cosmic age $t_{M\Lambda} = t\,(a_{M\Lambda}) = 9.5$ Gyr."

- p.405, Problem 13.11(a): Remove an extra factor of 1 in the subscript of the denominator factor g_{221} so that the displayed equation line reads as

$$\Gamma^2_{22} = \frac{1}{2g_{22}}\partial_2 g_{22}, \qquad \Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{2g_{22}}\partial_1 g_{22}$$

- p.405, Problem 13.11(a): Insert 3 minus signs in front of the last three right hand sides and remove an extra factor of ν in the subscript of $\Gamma^1_{1\nu 1}$ on the 3rd line so that the displayed equation reads as

$$\begin{aligned} & R_{1212} = g_{1\mu}R^\mu_{212} \\ = \; & -g_{11}\left(\partial_2\Gamma^1_{21} - \partial_1\Gamma^1_{22} + \Gamma^\nu_{21}\Gamma^1_{\nu 2} - \Gamma^\nu_{22}\Gamma^1_{\nu 1}\right) \\ = \; & -g_{11}(\partial_2\Gamma^1_{21} - \partial_1\Gamma^1_{22} + \Gamma^1_{21}\Gamma^1_{12} + \Gamma^2_{21}\Gamma^1_{22} - \Gamma^1_{22}\Gamma^1_{11} - \Gamma^2_{22}\Gamma^1_{21}) \\ = \; & -\frac{1}{2}\{\partial_2^2 g_{11} + \partial_1^2 g_{22} - \frac{1}{2g_{11}}\left[(\partial_1 g_{11})(\partial_1 g_{22}) + (\partial_2 g_{11})^2\right] \\ & -\frac{1}{2g_{22}}\left[(\partial_2 g_{11})(\partial_2 g_{22}) + (\partial_1 g_{22})^2\right]\} \end{aligned}$$

- p.405, Problem 13.11(a) and (b) the last five lines on p.405: Remove two minus signs: one in front of R_{1212}/g and the other in front of $2K$ so that these five lines now read

(a) The last two lines should read as ... "which, when divided by the metric determinant $g = g_{11}g_{22}$, the ratio R_{1212}/g is recognized as the Gaussian curvature of (5.35).

(b) The Ricci scalar is simply the twice contracted Riemann tensor $R = g^{\alpha\beta}g^{\mu\nu}R_{\alpha\mu\beta\nu} = 2g^{11}g^{22}R_{1212}$ because $R_{1212} = R_{2121}$. Since $g^{11} = 1/g_{11}$ and $g^{22} = 1/g_{22}$, $g^{11}g^{22} = 1/g$ for a diagonalized metric tensor. In this way the result (a) leads to $R = 2K$."

- p.406, Problem 13.11(c) the top line and the top displayed equation: Insert a minus sign in the inline equation $dA^2 = R^2{}_{112}A^1\sigma$, a minus sign after the 1st equal sign and remove a minus sign after the 2nd equal sign of the displayed equation so that the text and the displayed equation now read

(c) Eq (13.57) may be written as $dA^2 = -R^2{}_{112}A^1\sigma$. Since the angular excess is related to the vector component change as $\epsilon = dA^2/A^1$, we have

$$\begin{aligned} \epsilon &= -R^2{}_{112}\sigma = g^{22}R_{1212}\sigma = g^{22}gK\sigma \\ &= g^{22}\left(g_{11}g_{22}\right)K\sigma = K\sigma, \end{aligned}$$

- p.406, Problem 13.12(2) the right hand side of the last displayed equation: change two minus signs in the parentheses to plus signs so the displayed equation reads as

$$\begin{aligned} 0 &= [D_\lambda,[D_\mu,D_\nu]] + [D_\nu,[D_\lambda,D_\mu]] + [D_\mu,[D_\nu,D_\lambda]] \\ &= -D_\lambda R^\gamma{}_{\alpha\mu\nu}A_\gamma - D_\nu R^\gamma{}_{\alpha\lambda\mu}A_\gamma - D_\mu R^\gamma{}_{\alpha\nu\lambda}A_\gamma \\ &\quad +R^\gamma{}_{\lambda\mu\nu}D_\gamma A_\alpha + R^\gamma{}_{\nu\lambda\mu}D_\gamma A_\alpha + R^\gamma{}_{\mu\nu\lambda}D_\gamma A_\alpha \\ &= -\left(D_\lambda R^\gamma{}_{\alpha\mu\nu} + D_\nu R^\gamma{}_{\alpha\lambda\mu} + D_\mu R^\gamma{}_{\alpha\nu\lambda}\right)A_\gamma \\ &\quad +\left(R^\gamma{}_{\lambda\mu\nu} + R^\gamma{}_{\nu\lambda\mu} + R^\gamma{}_{\mu\nu\lambda}\right)D_\gamma A_\alpha. \end{aligned}$$

- p.411, solution to Problem 15.2, the equation at the bottom of the page: Insert a missing power of 3 over r in the very last factor so that the factor changes from "$\partial^\mu \bar{h}_{\mu\nu} = ... \quad = -\frac{x^i C_{i\nu}}{r} = 0$." to

$$\partial^\mu \bar{h}_{\mu\nu} = C_{\mu\nu}\partial^\mu\left(\frac{1}{r}\right) = C_{i\nu}\partial^i\left(\frac{1}{r}\right) = -\frac{x^i C_{i\nu}}{r^3} = 0.$$

- p.413, Problem 15.4(b): Insert the parenthetical remark so the first two lines of text just above the displayed equations will read as

(b) Ricci tensor: from what we know of Christoffel symbols having the nonvanishing elements of (after dropping higher order terms in $\tilde{h}_+$)