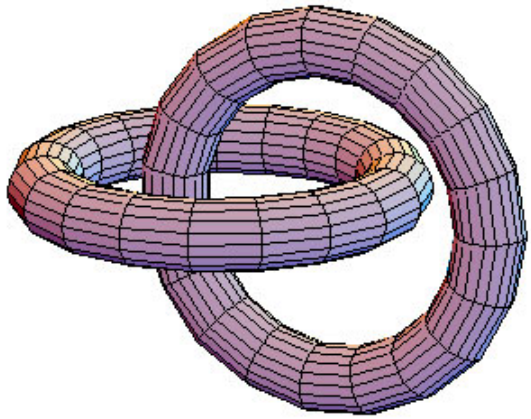

# Introduction to Tensor Calculus and Continuum Mechanics

*by J.H. Heinbockel*

*Department of Mathematics and Statistics*

*Old Dominion University*

## PREFACE

This is an introductory text which presents fundamental concepts from the subject areas of tensor calculus, differential geometry and continuum mechanics. The material presented is suitable for a two semester course in applied mathematics and is flexible enough to be presented to either upper level undergraduate or beginning graduate students majoring in applied mathematics, engineering or physics. The presentation assumes the students have some knowledge from the areas of matrix theory, linear algebra and advanced calculus. Each section includes many illustrative worked examples. At the end of each section there is a large collection of exercises which range in difficulty. Many new ideas are presented in the exercises and so the students should be encouraged to read all the exercises.

The purpose of preparing these notes is to condense into an introductory text the basic definitions and techniques arising in tensor calculus, differential geometry and continuum mechanics. In particular, the material is presented to (i) develop a physical understanding of the mathematical concepts associated with tensor calculus and (ii) develop the basic equations of tensor calculus, differential geometry and continuum mechanics which arise in engineering applications. From these basic equations one can go on to develop more sophisticated models of applied mathematics. The material is presented in an informal manner and uses mathematics which minimizes excessive formalism.

The material has been divided into two parts. The first part deals with an introduction to tensor calculus and differential geometry which covers such things as the indicial notation, tensor algebra, covariant differentiation, dual tensors, bilinear and multilinear forms, special tensors, the Riemann Christoffel tensor, space curves, surface curves, curvature and fundamental quadratic forms. The second part emphasizes the application of tensor algebra and calculus to a wide variety of applied areas from engineering and physics. The selected applications are from the areas of dynamics, elasticity, fluids and electromagnetic theory. The continuum mechanics portion focuses on an introduction of the basic concepts from linear elasticity and fluids. The Appendix A contains units of measurements from the Système International d'Unitès along with some selected physical constants. The Appendix B contains a listing of Christoffel symbols of the second kind associated with various coordinate systems. The Appendix C is a summary of useful vector identities.

J.H. Heinbockel, 1996

# INTRODUCTION TO TENSOR CALCULUS AND CONTINUUM MECHANICS

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