

# Research on the Theory of the TeV energy scale (Terascale)

Howard Haber

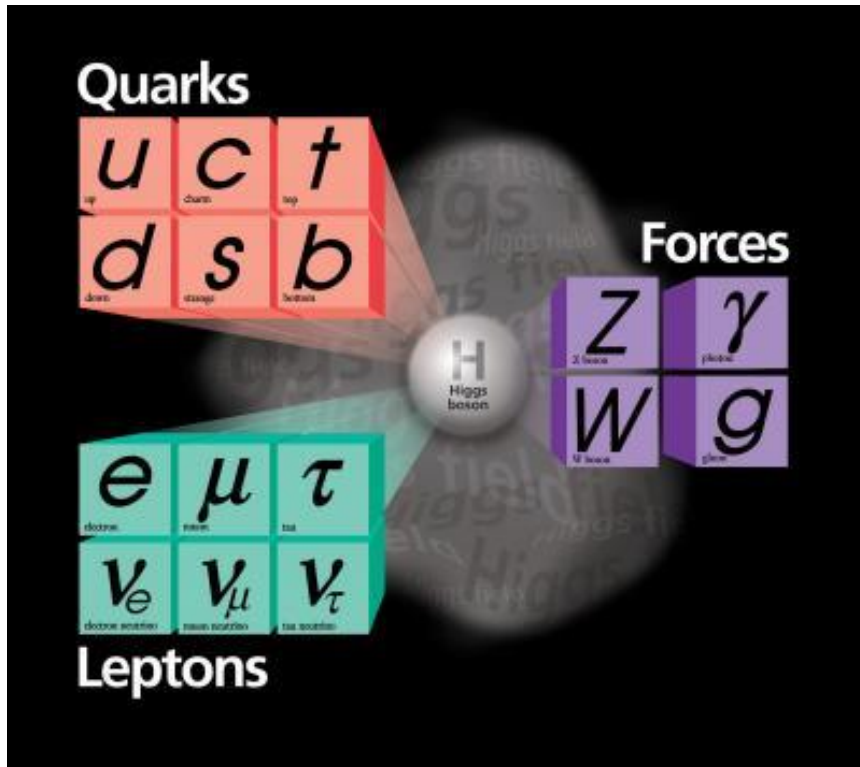
SCIPP Theory

March 5, 2026

For further details, check out my webpage:

<https://scipp-legacy.pbsci.ucsc.edu/~haber/>

# The Standard Model (SM) of Particle Physics



The elementary particles consists of three generations of spin-1/2 quarks and leptons, the gauge bosons of  $SU(3) \times SU(2) \times U(1)$ , and the Higgs boson.

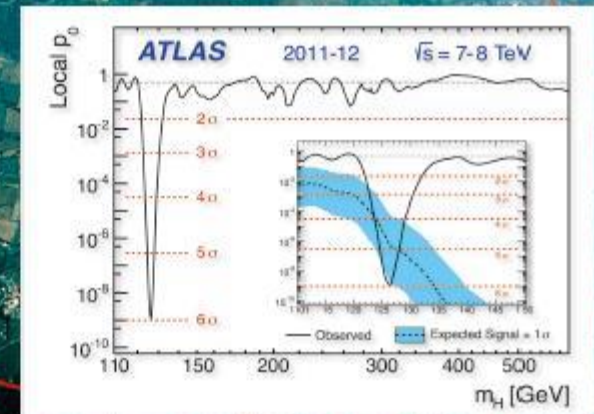
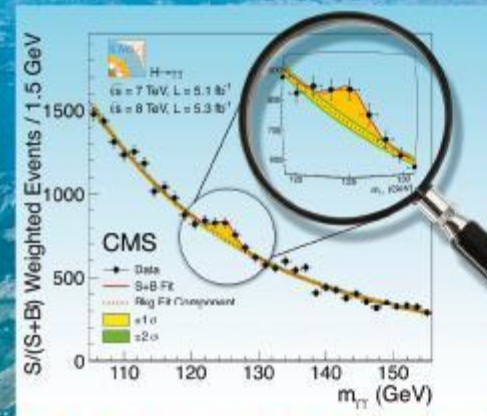
Technically, massive neutrinos require an extension of the Standard Model, but most likely the relevant scale of the new physics lies way beyond the terascale.



# PHYSICS LETTERS B

Available online at [www.sciencedirect.com](http://www.sciencedirect.com)

SciVerse ScienceDirect



On July 4, 2012, the discovery of a new boson is announced which may be the long sought after Higgs boson.

The discovery papers are published two months later In Physics Letters B.

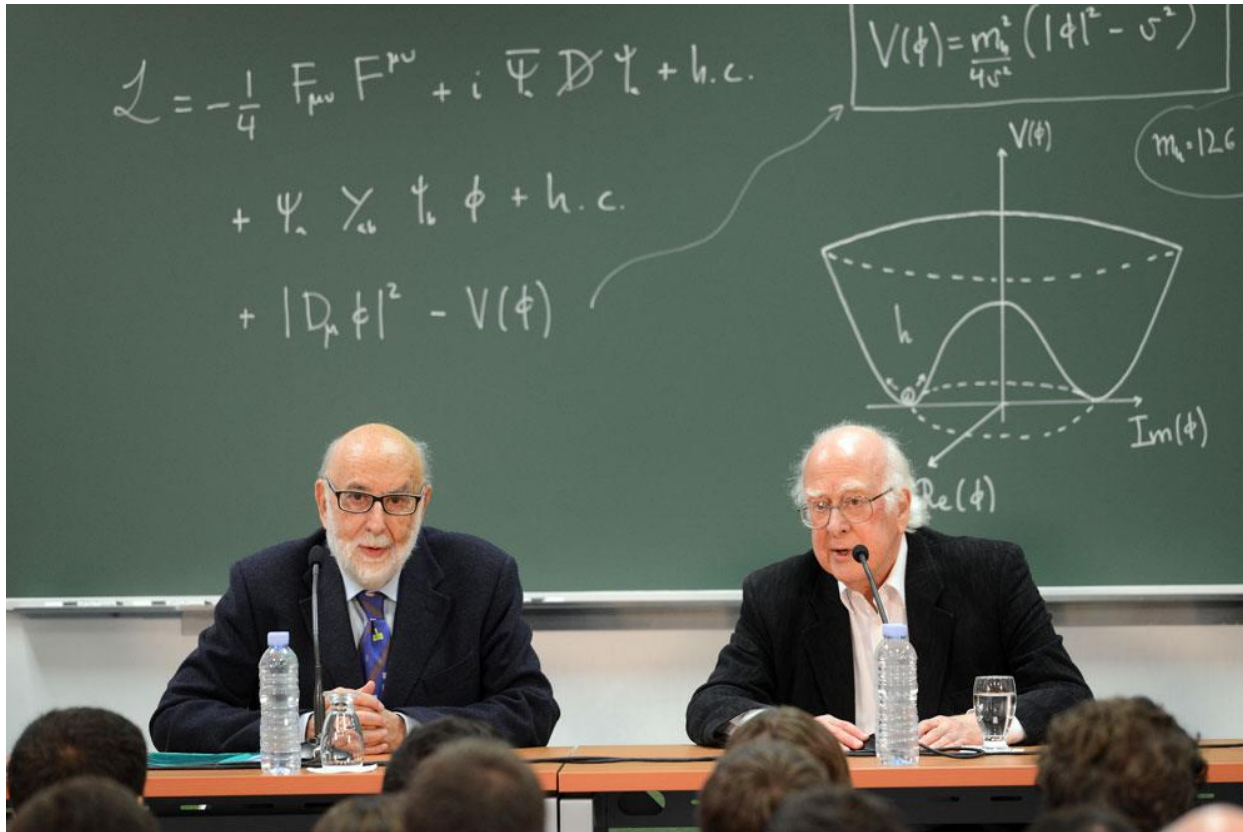
ATLAS Collaboration:

**Physics Letters B716 (2012) 1—29**

CMS Collaboration:

**Physics Letters B716 (2012) 30—61**

# Winners of the 2013 Nobel Prize in Physics

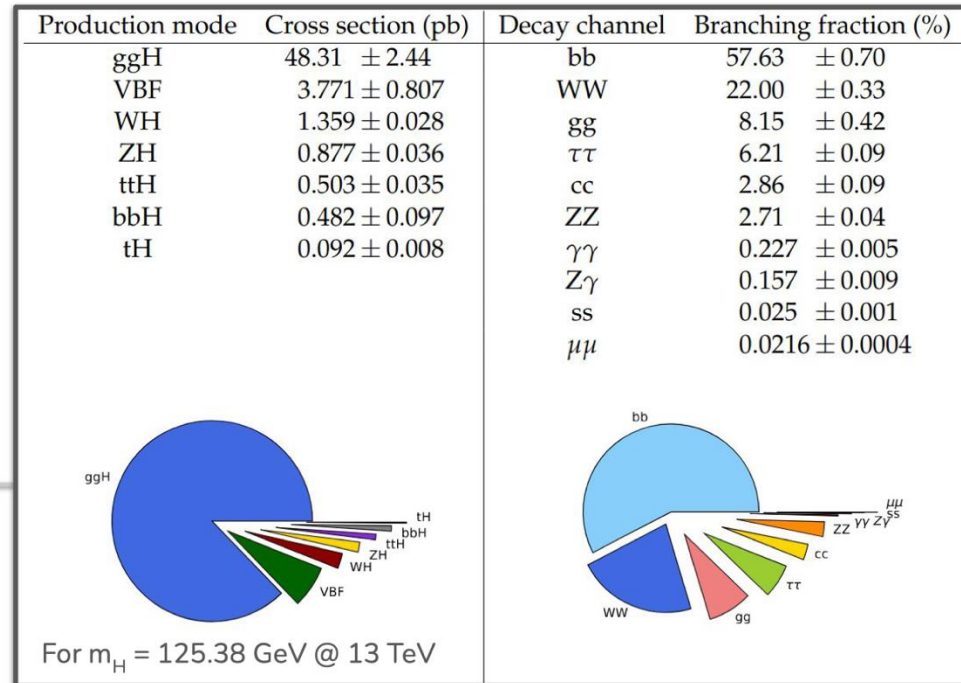
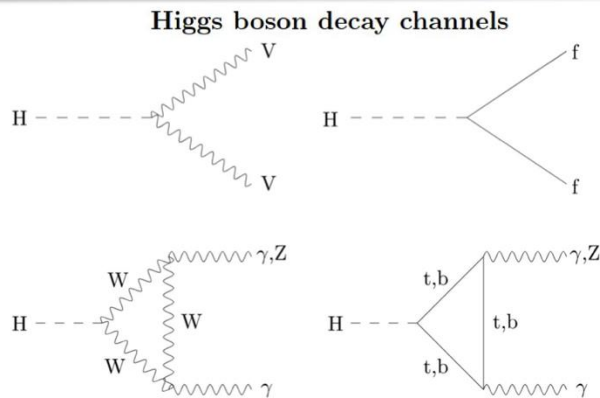
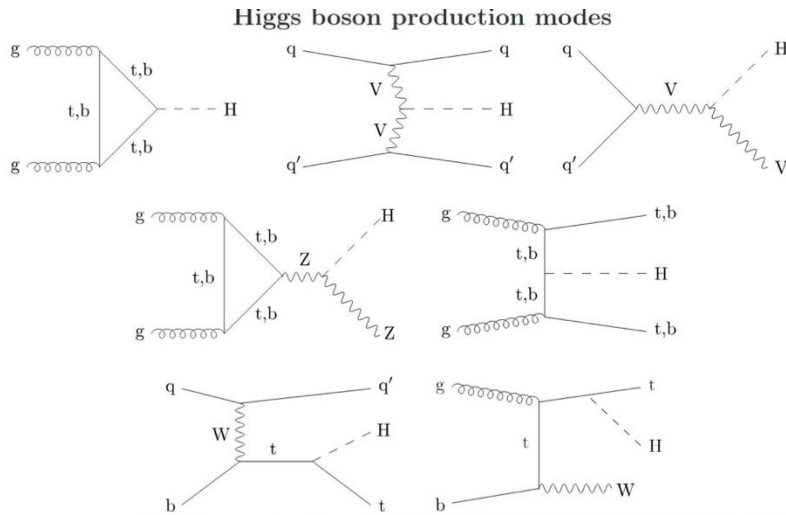


**François Englert**

and

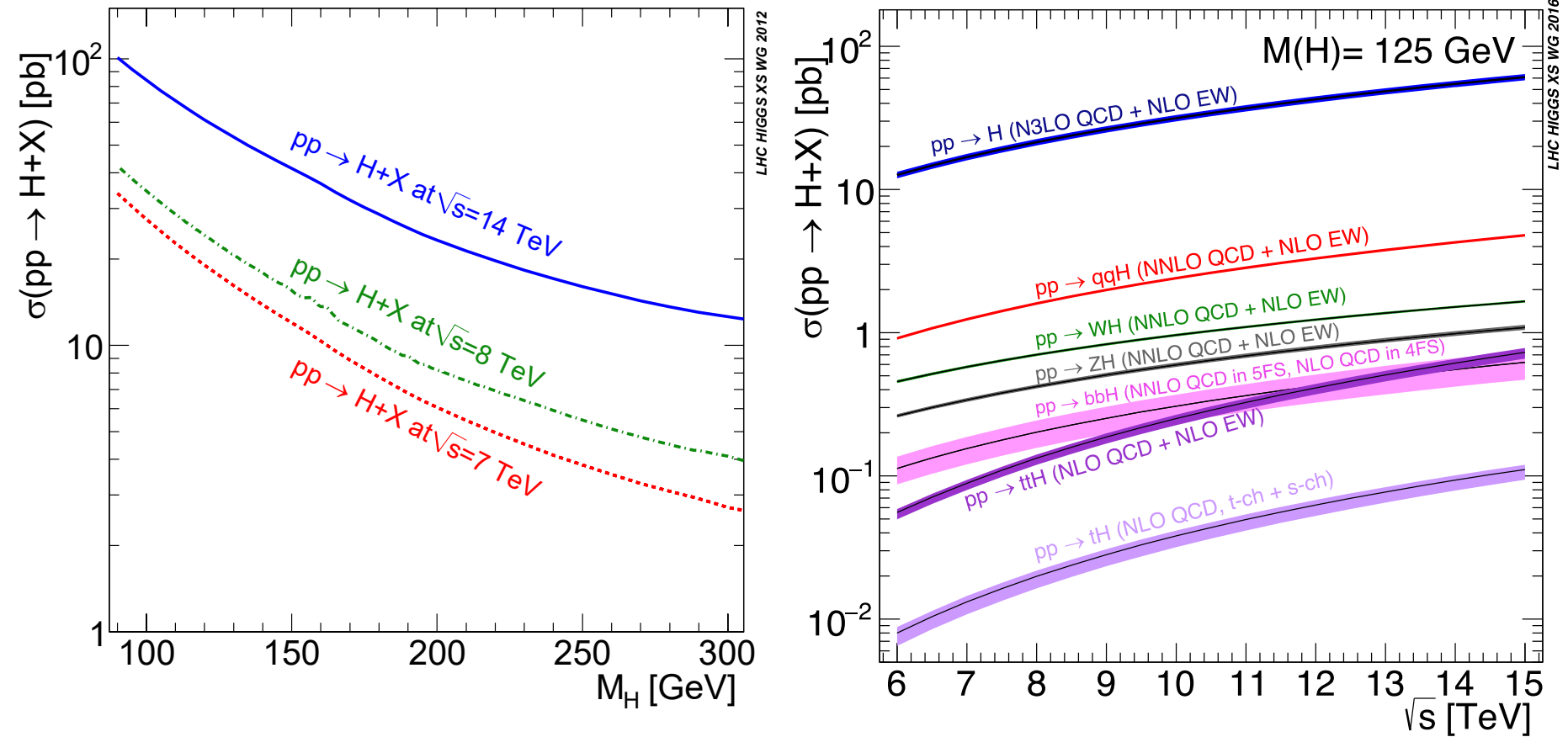
**Peter Higgs**

# Higgs boson production and decay mechanisms



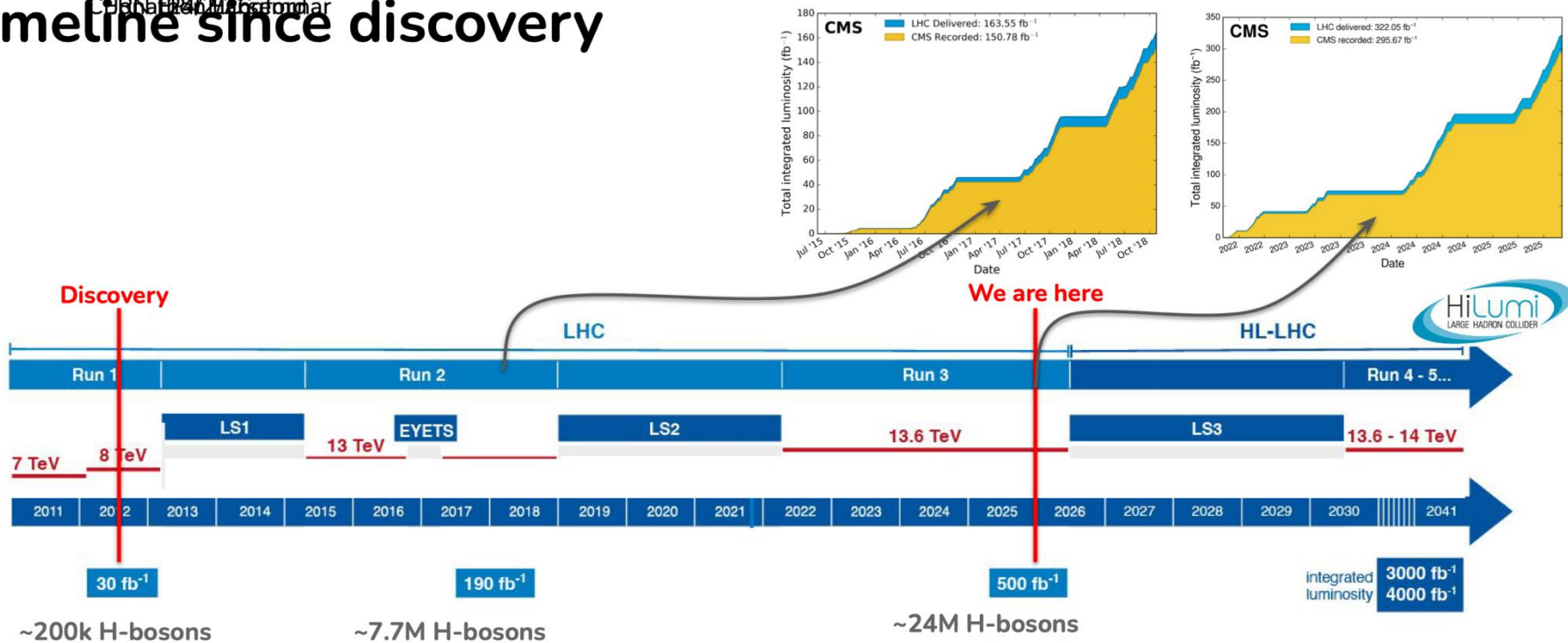
- Theory community worked tirelessly to understand these interactions and the associated kinematics
- **Precise reference to compare to experimental measurements**

# Higgs boson production cross sections at a pp collider



With nearly  $140 \text{ fb}^{-1}$  of data delivered by the LHC in Run 2 to both ATLAS and CMS in 2015—2018 at a center of mass energy of 13 TeV, roughly 7.7 million Higgs bosons per experiment were produced, assuming the Higgs mass is 125 GeV.

# Timeline since discovery

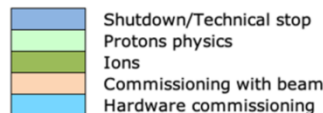
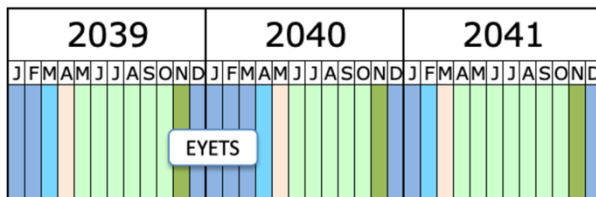
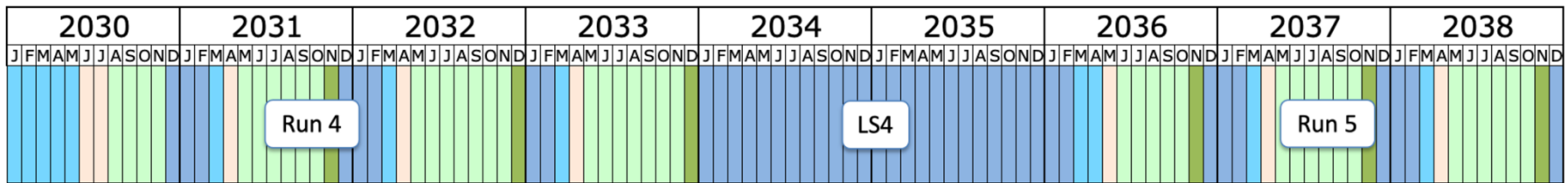
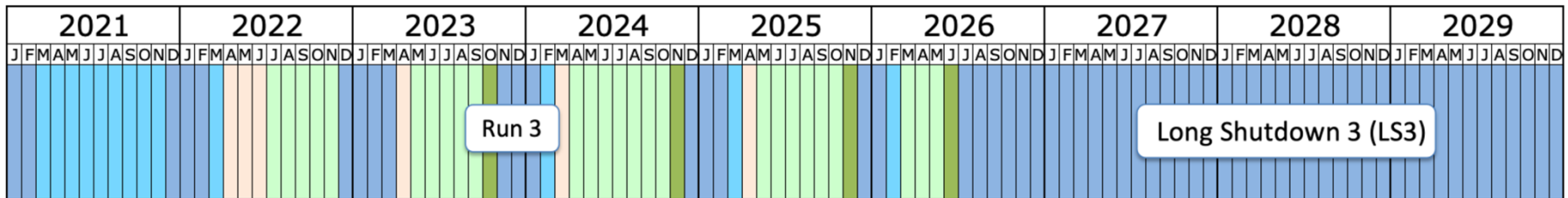


Taken from a 02/24/2026 seminar by Jonathon Langford on behalf of the CMS Collaboration

# Long Term LHC Schedule (to 2041): Update Sept. 2024

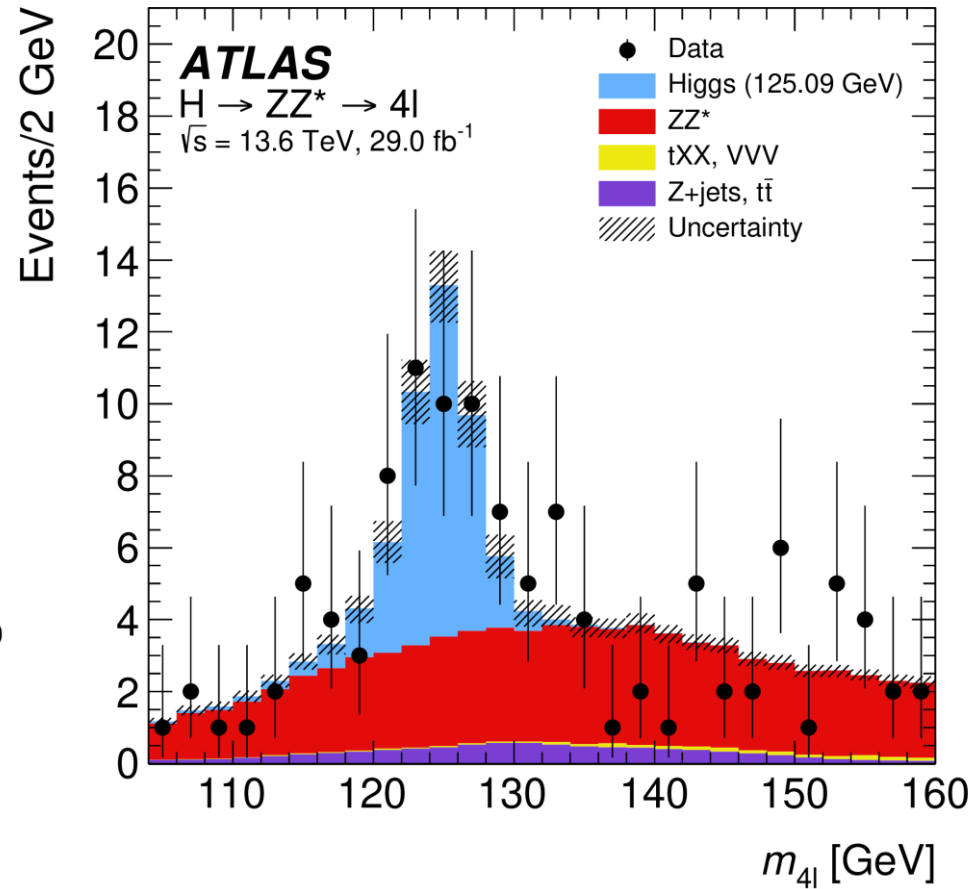
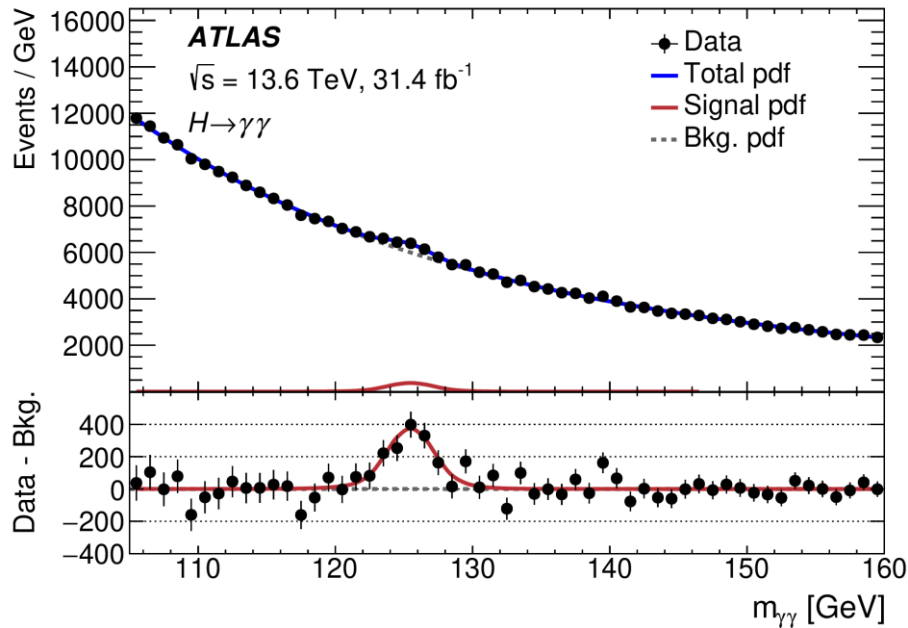
**Towards HL LHC:  
 Challenge and  
 Opportunity**

- Short YETS 25/26
- Extend Run 3 to end June 2026
- Start LHC LS3 July 2026
- Start final Hardware Commissioning January 2030
- First beam June 2030
- LS3 - beam to beam: 3 years 11 months, 47 months



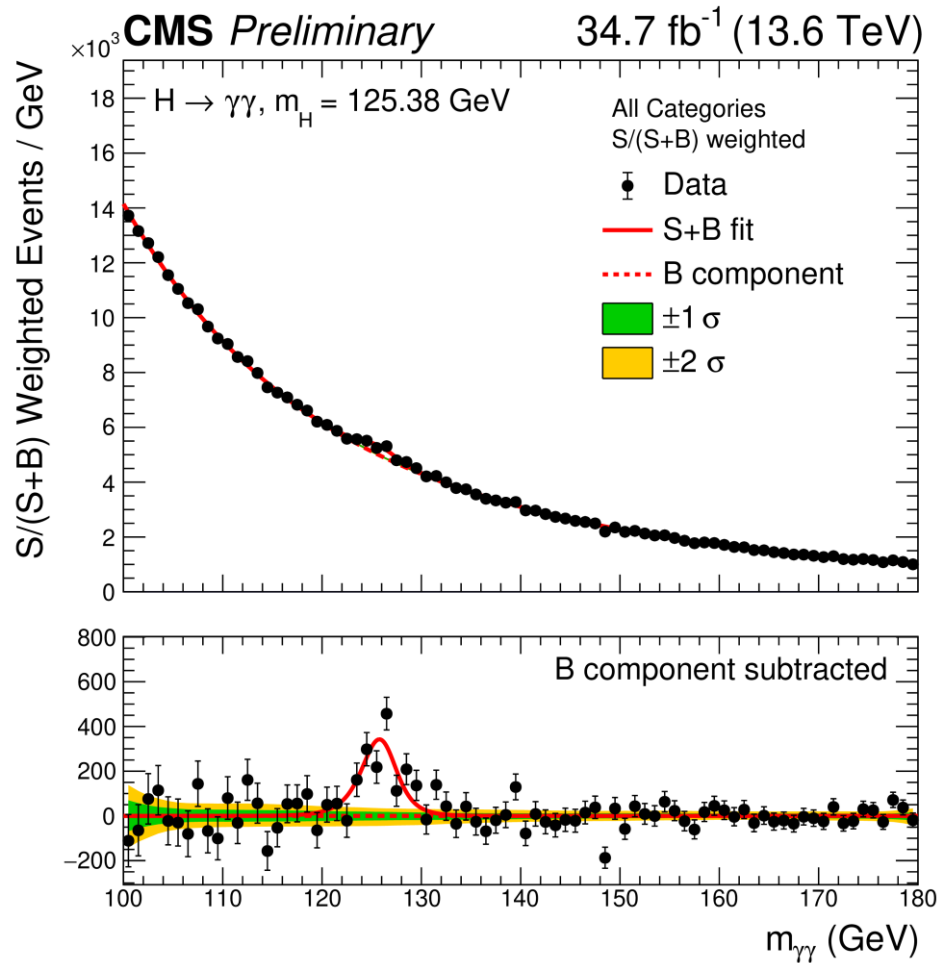
- Looking forward to a massive (~380/fb) Run3 dataset by end 2026
- A challenge to the LHC Phase 1 accelerator and experiment designs

# ATLAS Run 3 observations of the Higgs boson

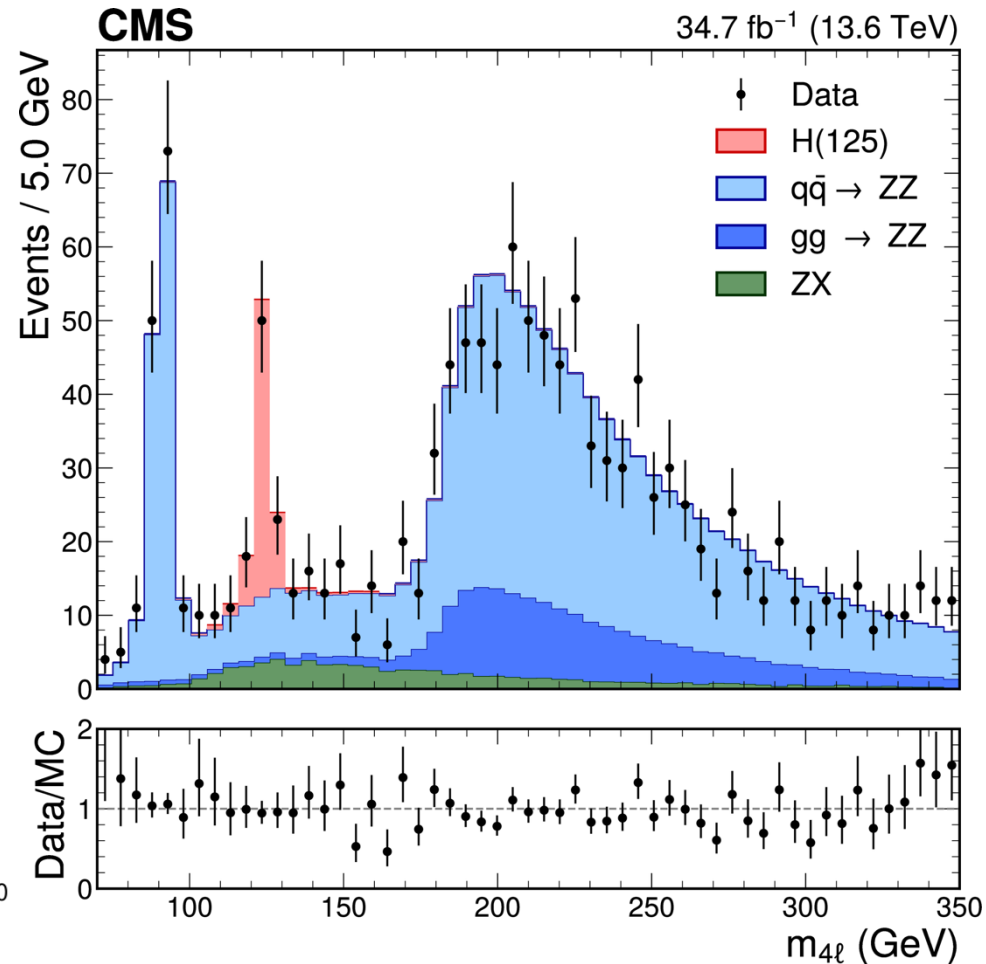


Taken from [Eur. Phys. J. C 84 \(2024\) 78](#)

# CMS Run 3 observations of the Higgs boson



Taken from CMS-PAS-HIG-23-014



Taken from CERN-EP-2024-336

# Higgs boson decay channels observed at the LHC

Higgs boson decay mode	Branching ratio (for $m_h = 125 \text{ GeV}$ )
$h^0 \rightarrow \mathbf{bb}$	0.582
$h^0 \rightarrow \mathbf{\tau^+ \tau^-}$	$6.27 \times 10^{-2}$
$h^0 \rightarrow \mathbf{\ell^+ \ell^- \nu \nu}$ ( $\ell = \mathbf{e}$ or $\mathbf{\mu}$ )	$1.06 \times 10^{-2}$
$h^0 \rightarrow \mathbf{\gamma \gamma}$	$2.27 \times 10^{-3}$
$h^0 \rightarrow \mathbf{\ell^+ \ell^- \ell^+ \ell^-}$ ( $\ell = \mathbf{e}$ or $\mathbf{\mu}$ )	$1.24 \times 10^{-4}$
$h^0 \rightarrow \mathbf{Z \gamma} \rightarrow \mathbf{\ell^+ \ell^- \gamma}$ ( $\ell = \mathbf{e}$ or $\mathbf{\mu}$ )	$1.03 \times 10^{-4}$
$h^0 \rightarrow \mathbf{\mu^+ \mu^-}$	$2.18 \times 10^{-4}$

Taken from [https://twiki.cern.ch/twiki/bin/view/LHCPhysics/CERNYellowReportPageBR#Branching\\_Ratios](https://twiki.cern.ch/twiki/bin/view/LHCPhysics/CERNYellowReportPageBR#Branching_Ratios)

## Remarks:

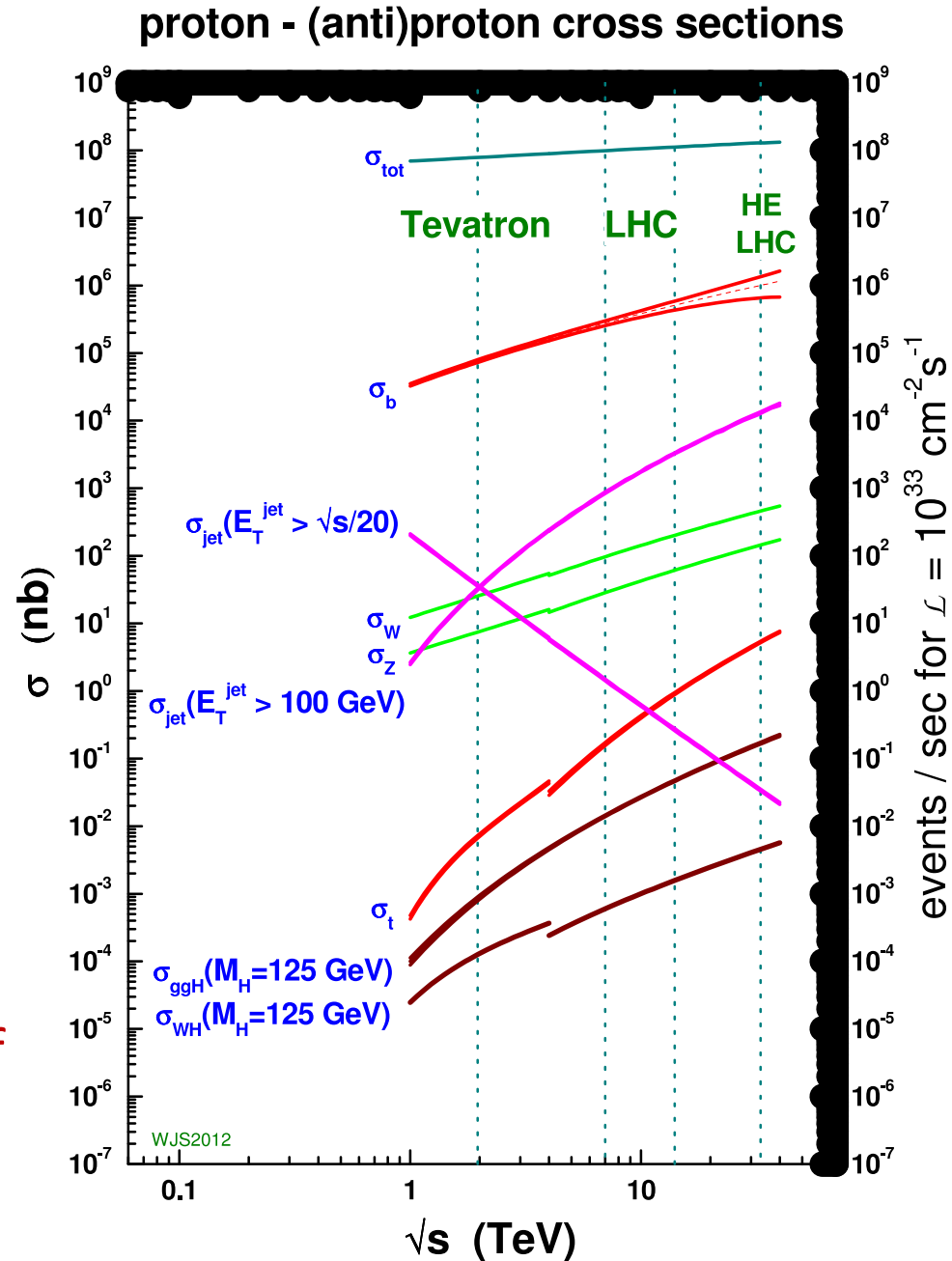
1.  $h^0 \rightarrow WW^*$  is observed primarily via the  $\ell^+ \nu \ell^- \nu$  ( $\ell = \mathbf{e}$  or  $\mathbf{\mu}$ ) final state.
2.  $h^0 \rightarrow ZZ^*$  is observed primarily via the  $\ell^+ \ell^- \ell^+ \ell^-$  ( $\ell = \mathbf{e}$  or  $\mathbf{\mu}$ ) final state.

In the decays to the diboson final state, kinematics dictates that one of the vector bosons is off-shell (i.e., “virtual”) and is thus indicated by a superscript star.

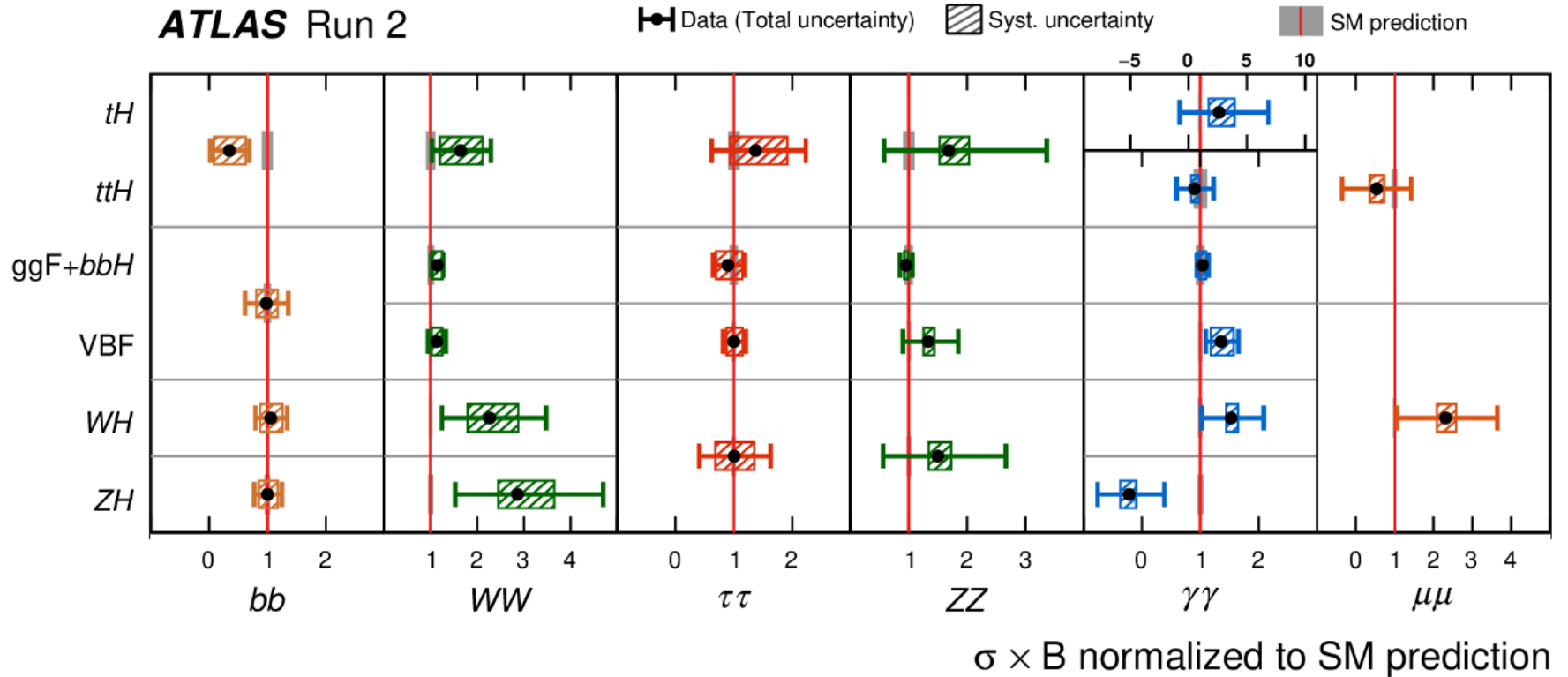
Question: why not search inclusively for Higgs bosons that decay into a pair of b-quarks?

Answer: The Standard Model background is overwhelming. There are more than  $10^7$  times as many b-quark pairs produced in proton-proton collisions as compared to b-quark pairs that arise from a decaying Higgs boson.

Nevertheless, the observation of  $H \rightarrow bb$  in the VH channel was confirmed by ATLAS and CMS in 2018!

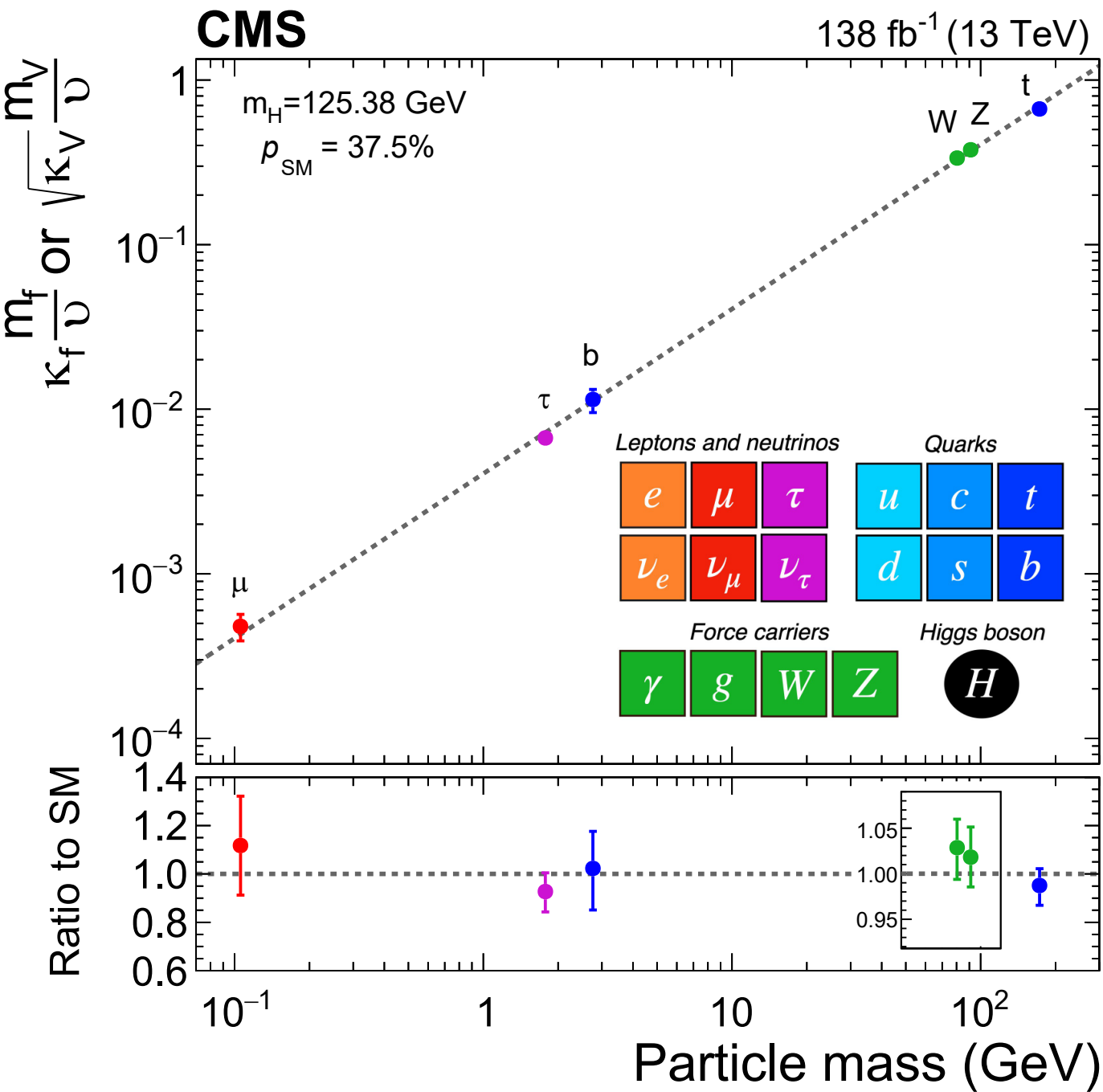


# Summary of ATLAS Higgs boson data from Run 2 at the LHC



**Fig. 3 | Ratio of observed rate to predicted standard model event rate for different combinations of Higgs boson production and decay processes.** The horizontal bar on each point denotes the 68% confidence interval. The narrow grey bands indicate the theory uncertainties in the standard model

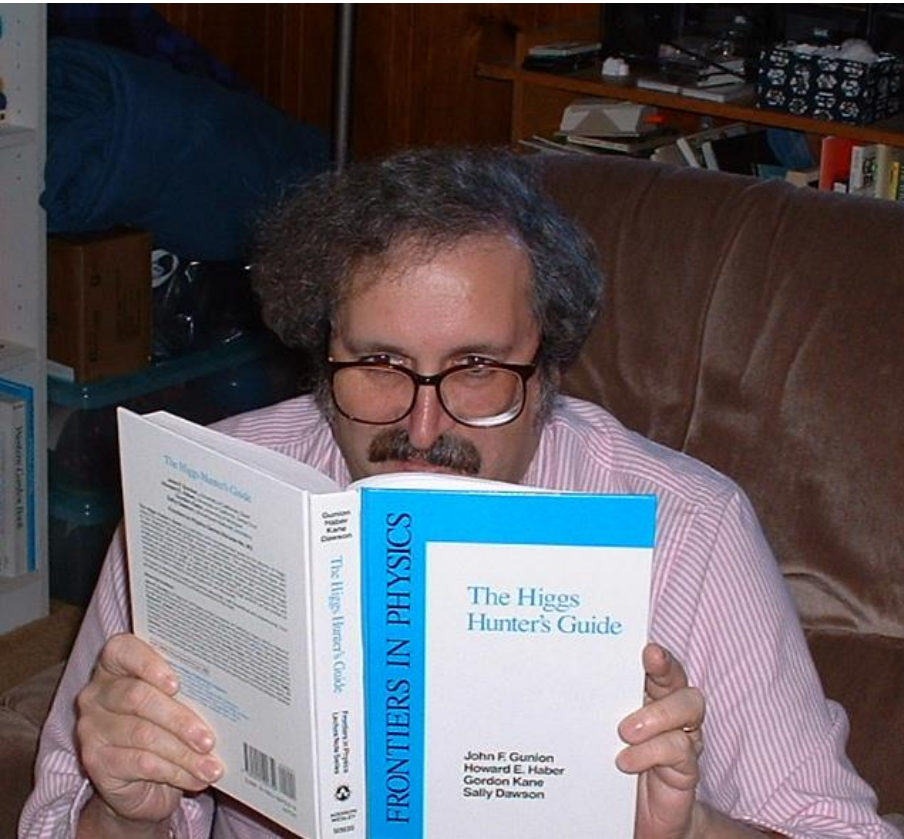
(SM) cross-section multiplied by the branching fraction predictions. The  $p$  value for compatibility of the measurement and the SM prediction is 72%.  $\sigma_i B_i$  is normalized to the SM prediction. Data are from ATLAS Run 2.



Reduced Higgs coupling modifiers compared to their corresponding prediction from the Standard Model (SM). The error bars represent 68% CL intervals for the measured parameters. In the lower panel, the ratios of the measured coupling modifiers values to their SM predictions are shown.

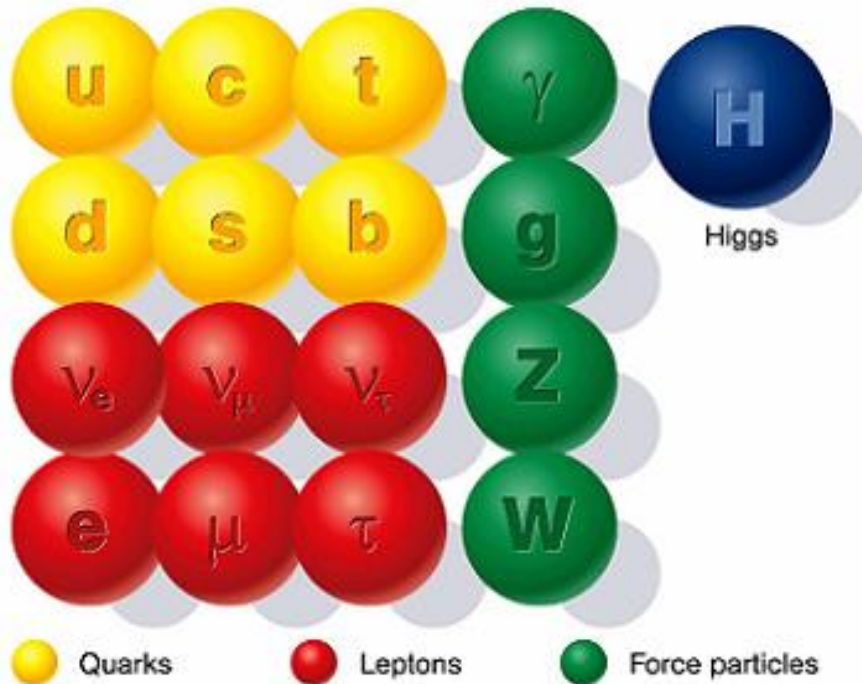
[Taken from: Nature 607 (2022) 60]

# Research program 1: theory and phenomenology of Higgs bosons

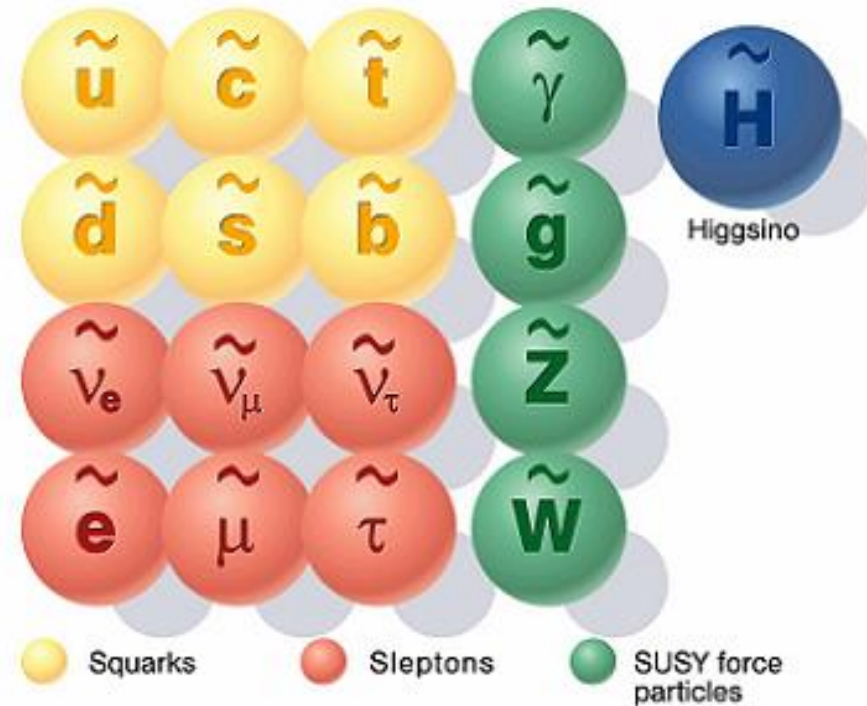


# Research program 2: theory and phenomenology of TeV-scale supersymmetry (SUSY)

## Standard particles



## SUSY particles



# As members of the Particle Data Group, B.C. Allanach and I are co-authors of the biennial Supersymmetry Theory review.

## PHYSICAL REVIEW D

Articles Published in AUGUST 2024  
DI  
PART A

covering particles, fields,  
gravitation, and cosmology

### Review of Particle Physics

S. Navas et al. (Particle Data Group), Phys. Rev. D 110, 030001 (2024)

**PDG**  
particle data group

Published by  
AMERICAN PHYSICAL SOCIETY

**APS**

Volume 110

Third Series

Number 3

1038 88. Supersymmetry, Part I (Theory)

### 88. Supersymmetry, Part I (Theory)

Revised August 2023 by B.C. Allanach (DAMTP, Cambridge U.) and H.E. Haber (UC Santa Cruz).

88.1	Introduction . . . . .	1038
88.2	Structure of the MSSM . . . . .	1038
88.2.1	R-parity and the lightest supersymmetric particle . . . . .	1039
88.2.2	The goldstino and gravitino . . . . .	1039
88.2.3	Hidden sectors and the structure of SUSY breaking . . . . .	1040
88.2.4	SUSY and extra dimensions . . . . .	1040
88.2.5	Split-SUSY . . . . .	1040
88.3	Parameters of the MSSM . . . . .	1041
88.3.1	The SUSY-conserving parameters . . . . .	1041
88.3.2	The SUSY-breaking parameters . . . . .	1041
88.3.3	MSSM-124 . . . . .	1041
88.4	The supersymmetric-particle spectrum . . . . .	1042
88.4.1	The charginos and neutralinos . . . . .	1042
88.4.2	The squarks and sleptons . . . . .	1043
88.5	The supersymmetric Higgs sector . . . . .	1043
88.5.1	The tree-level Higgs sector . . . . .	1043
88.5.2	The radiatively-corrected Higgs sector . . . . .	1044
88.6	Restricting the MSSM parameter freedom . . . . .	1044
88.6.1	Gaugino mass relations . . . . .	1045
88.6.2	Constrained versions of the MSSM: mSUGRA, CMSSM, etc. . . . .	1046
88.6.3	Gauge-mediated SUSY breaking . . . . .	1046
88.6.4	The phenomenological MSSM . . . . .	1047
88.6.5	Simplified models . . . . .	1047
88.7	Experimental data confronts the MSSM . . . . .	1047
88.7.1	Naturalness constraints and the little hierarchy . . . . .	1048
88.7.2	Indirect constraints on supersymmetric models . . . . .	1049
88.8	Massive neutrinos in weak-scale SUSY . . . . .	1050
88.8.1	The supersymmetric seesaw . . . . .	1050
88.8.2	R-parity-violating SUSY . . . . .	1050
88.9	Extensions beyond the MSSM . . . . .	1051

#### 88.1 Introduction

Supersymmetry (SUSY) is a generalization of the space-time symmetries of quantum field theory that transforms fermions into bosons and vice versa [1]. The existence of such a non-trivial extension of the Poincaré symmetry of ordinary quantum field theory was initially surprising, and its form is highly constrained by theoretical principles [2]. SUSY also provides a framework for the unification of particle physics and gravity [3–5] at the Planck energy scale,  $M_P \sim 10^{19}$  GeV, where the gravitational interactions become comparable in strength to the gauge interactions.

less, with some restrictions on the dimension-three terms added in Ref. [10]. The impact of the soft terms becomes at energy scales much larger than the size of the SUSY masses. Thus, a theory of weak-scale supersymmetry effective scale of supersymmetry breaking is tied to electroweak symmetry breaking, provides a natural framework for the origin and the stability of the gauge hierarchy [6–9].

At present, there is no unambiguous experimental evidence for the breakdown of the SM at or below the TeV scale. Predictions for new TeV-scale physics beyond the SM are primarily on three theoretical arguments. First, in a minimal supersymmetric extension of the SM, the production of an elementary scalar field of mass  $m$  and interaction strength  $\lambda$  (e.g., a quartic scalar self-coupling, the square of a gauge coupling, or the square of a Yukawa coupling), the stability of the vacuum requires quantum corrections requires the existence of an energy scale of order  $(16\pi^2/\lambda)^{1/2}m$ , beyond which new physics must appear [12]. A significantly larger energy cutoff would require unnatural fine-tuning of parameters that govern the effective theory. Applying this argument to the SM expectation of new physics at the TeV scale [9].

Second, the unification of the three SM gauge couplings at a very high energy close to the Planck scale is possible if there is new physics beyond the SM (which modifies the running of the gauge couplings) above the electroweak scale) is present. The minimal supersymmetric extension of the SM, where superpartner masses are a few TeV, provides an example of successful gauge coupling unification [13].

Third, the existence of dark matter that makes up approximately one quarter of the energy density of the universe can be explained within the SM of particle physics [14]. If there is a stable weakly-interacting massive particle (WIMP) with a production and interaction rate are governed by new physics at the TeV-scale can be consistent with the observed dark matter density (this is the so-called WIMP miracle, which is discussed in Ref. [15]). The lightest supersymmetric particle (LSP) is a promising (although not the unique) candidate for dark matter [16–20]. Further aspects of dark matter can be found in Sec. 27.

#### 88.2 Structure of the MSSM

The minimal supersymmetric extension of the SM (MSSM) consists of the fields of the two-Higgs-doublet extension of the corresponding superpartner fields [21–25]. A part of the superpartner together form a supermultiplet. The field content of the supermultiplets of the MSSM and quantum numbers are shown in Table 88.1. The electric charge  $Q = T_3 + \frac{1}{2}Y$  is determined in terms of the third component of the weak isospin ( $T_3$ ) and the U(1) weak hypercharge.

The gauge supermultiplets consist of the gluons and fermionic superpartners and the SU(2)×U(1) gauge theory supermultiplets consist of the gluons and fermionic superpartners. The matter supermultiplets consist of three generations of left-handed quarks

2025 update available at <https://pdg.lbl.gov/>

Published by Cambridge University Press on June 7, 2023

# From Spinors to Supersymmetry

Dreiner, Haber  
and Martin

## From Spinors to Supersymmetry

Herbi K. Dreiner, Howard E. Haber  
and Stephen P. Martin

The new book by Dreiner, Haber, and Martin is a must have for folks who are interested in beyond the Standard Model phenomenology. It contains innumerable lessons for performing quantum field theory calculations both at the conceptual and technical level, by way of many concrete examples within the Standard Model and its supersymmetric extension. I expect this will become a go-to reference for everyone from graduate students to seasoned researchers.”  
Prof. Tim Cohen, CERN/EPFL and the University of Oregon

The book gives a self-contained description of the Standard Model of particle physics and its supersymmetric extension. It is well suited for students, as well as experienced researchers in the field. Its unique feature is the comprehensive description of quantum field theory and its application to particle physics in the framework of two-component (Weyl) spinors. [...] The book will be of enormous help to all those that try to teach and try to learn the subject.”  
Prof. Hans-Peter Nilles, Universität Bonn

This is a massive, definitive text on phenomenological supersymmetry in quantum field theory by three giants of the field. The book develops two-component spinor formalism and its practical use in amplitude computations with many phenomenological examples up to one loop order. Supersymmetric extensions of the Standard Model are also covered and many other gems besides.”  
Prof. Ben Allanach, University of Cambridge

Supersymmetry is an extension of the successful Standard Model of particle physics; it relies on the principle that fermions and bosons are related by a symmetry, leading to an elegant predictive structure for quantum field theory. This textbook provides a comprehensive and pedagogical introduction to supersymmetry and other aspects of particle physics at the high-energy frontier. Aimed at graduate students and researchers, it also discusses concepts of physics beyond the Standard Model, including extended Higgs sectors, grand unification, and the origin of neutrino masses.

Cover image: Sierralara/Room/Getty Images

 CAMBRIDGE  
UNIVERSITY PRESS

ISBN 978-0-521-80088-4



9 780521 800884

CAMBRIDGE

Designed by EMC Design Ltd



Cruz

UCSC

UCSC

From Spinors to Supersymmetry

Herbi K. Dreiner, Howard E. Haber, and Stephen P. Martin

## Research program 3: explorations of the Terascale at the LHC and at future colliders

- Studies of non-minimal Higgs sectors
- Precision measurements of new physics observables
- Distinguishing among different theoretical interpretations of new physics signals
- Using a future lepton collider as a precision Higgs factory
- Terascale footprints of lepton-number-violation
- New sources for CP-violation (Higgs and/or SUSY mediated)

## Publications (2024—2026)

### Supersymmetry, Part I (Theory)--2025 update

B. Allanach and H.E. Haber, [arXiv:2401.03827v3](#) [hep-ph]

### Correlating Resonant Di-Higgs and Tri-Higgs Production to $H \rightarrow VV$ in the 2HDM

G. Coloretti, A. Crivellin, and H.E. Haber, [arXiv:2512.24868](#) [hep-ph].

### Extending the symmetries of the generalized CP-symmetric 2HDM scalar potential to the Yukawa sector

S. Carrolo, H.E. Haber, L. Lourenco and J.P. Silva, Phys. Rev. D **112**, 035024 (2025).

### RG-stable parameter relations of a scalar field theory in absence of a symmetry

H.E. Haber, and P. Ferreira, Eur. Phys. J. C **85**, 541 (2025) [Erratum: **85**, 867 (2025)].

### Correlating $A \rightarrow \gamma\gamma$ with EDMs in the 2HDM in light of the diphoton excesses at 95 GeV and 152 GeV

S. Banik, G. Coloretti, A. Crivellin, and H.E. Haber, Phys. Rev. D **111**, 075021 (2025).

### Explicit form for the most general Lorentz transformation revisited

H.E. Haber, Symmetry 2024, 16, 1155.

### Classes of complete dark photon models constrained by Z-Physics

M. Bento, H.E. Haber, and J.P. Silva, Phys. Lett. B **850**, 138501 (2024).

# Major thrusts in phenomenological particle physics today

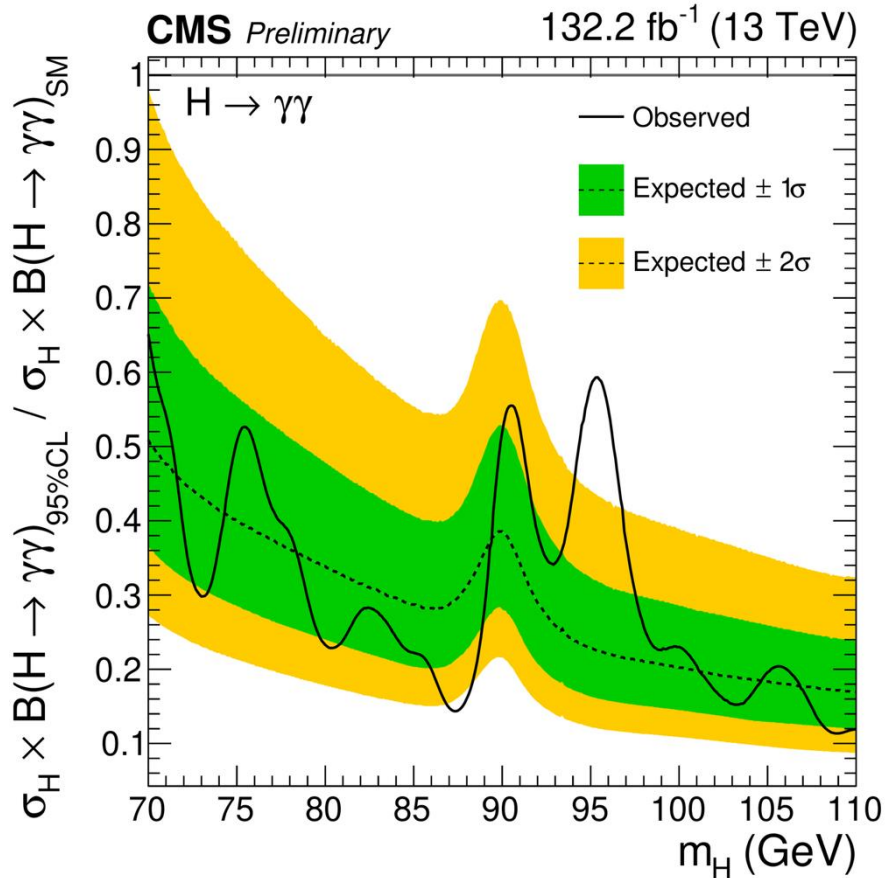
## ➤ What lies beyond the Standard Model and why haven't we seen it yet?

- New physics beyond the Standard Model (BSM) may be associated with a new heavy mass scale of order a few TeV or larger. If accessible at the LHC, not enough events have been produced yet (more luminosity needed). If the LHC is not energetic enough, one would need a higher energy collider facility.
- New BSM physics may be very weakly coupled to the Standard Model (SM). It could consist of completely new sectors of particles (e.g., the dark sector). The origin of dark matter could reside here. Many possibilities exist, so it is difficult to guess where the breakthrough will occur.
- If new BSM physics is completely neutral with respect to the SM, then it can only communicate with the SM via “portals” that consist of products of SM fields that have no net SM (color, weak or EM) charge.  
Examples: the Higgs portal  $H^\dagger H$ ; the neutrino portal  $H^\dagger L N$  ( $N$  could be a new sterile neutrino); or photon mixing  $F_{\mu\nu} X^{\mu\nu}$  (where  $X$  is the dark photon).

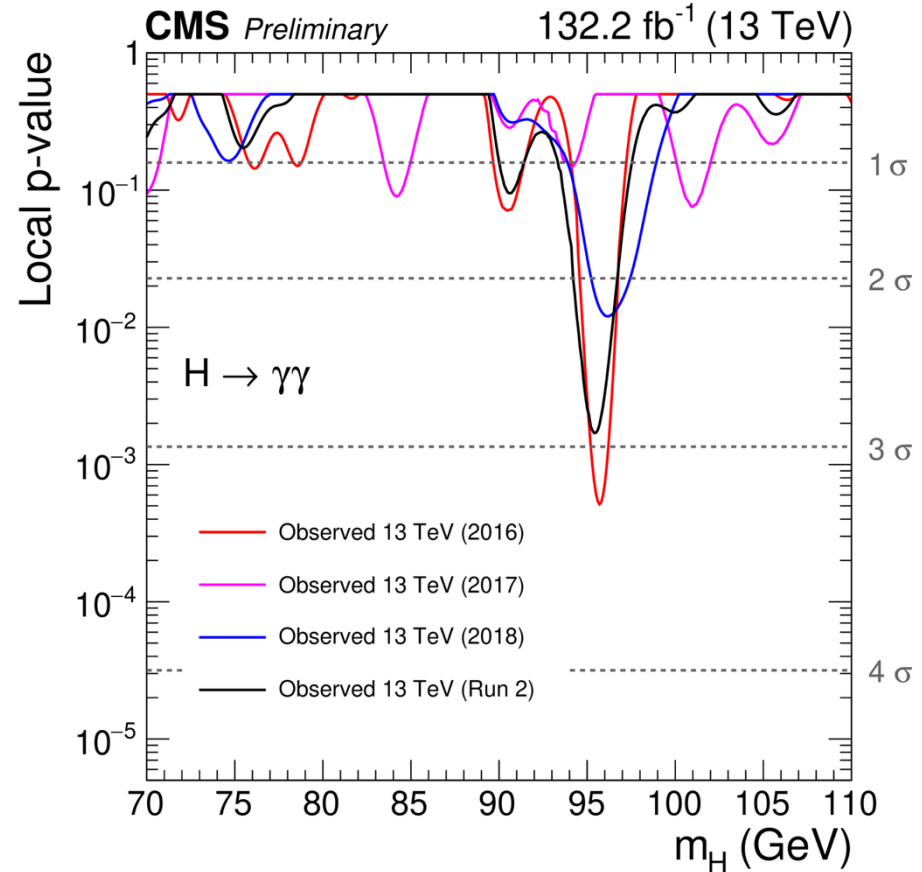
# Should we expect an extended Higgs sector beyond the SM?

- The fermion and gauge boson sectors of the SM are not of minimal form (“who ordered that?”). So, why should the spin-0 (scalar) sector be minimal?
- Adding new scalar states can alleviate the metastability of the vacuum, allowing the Higgs-sector-extended SM to be valid all the way up to the Planck scale.
- Extended Higgs sectors can provide a dark matter candidate.
- Extended Higgs sectors can provide new sources of CP violation (which may be useful in baryogenesis).
- Models of physics beyond the SM often require additional scalar Higgs states. E.g., two Higgs doublets are required in the minimal supersymmetric extension of the SM (MSSM).

# Evidence for a new Higgs scalar ?



Expected and observed exclusion limits (95% CL, in the asymptotic approximation) on the product of the production cross section and branching fraction into two photons for an additional SM-like Higgs boson, from the analysis of the combined data from 2016, 2017, and 2018. The inner and outer bands indicate the regions containing the distribution of limits located within  $\pm 1$  and  $2\sigma$ , respectively, of the expectation under the background-only hypothesis.

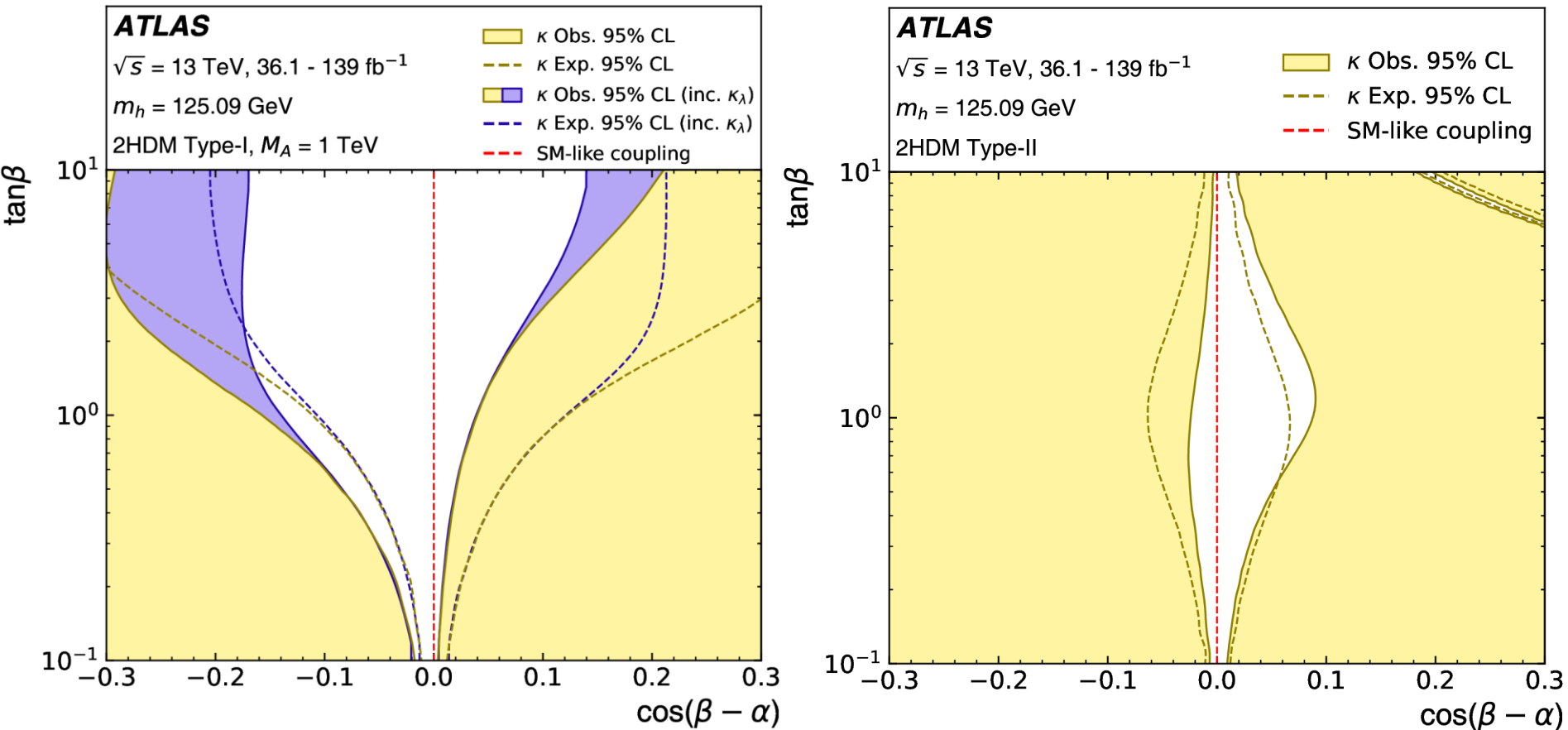


The observed local  $p$ -values for an additional SM-like Higgs boson as a function of  $m_H$ , from the analysis of the data from 2016, 2017, 2018, and their combination. Taken from CMS-PAS-HIG-20-002 (20 March 2023).

# Why is the observed Higgs boson SM-like?

- There is no extended Higgs sector.
- All other scalars (apart from the SM-like Higgs boson) are very heavy
  - This is the decoupling limit.
- A neutral scalar field with the tree-level properties of the SM Higgs boson is an approximate mass eigenstate (due to suppressed mixing with other neutral scalar fields of the extended Higgs sector).
  - This is the Higgs field alignment limit.
  - The other physical scalars of the model may or may not be significantly heavier than the SM Higgs boson. That is, the decoupling limit is a special case of the Higgs field alignment limit.

# Experimental constraints on the two Higgs doublet model (2HDM)



Regions excluded at 95% CL in the  $\kappa$ -framework-based approach by the measured rates of Higgs boson production and decays in the 2HDM with Type-I and Type-II Yukawa couplings, respectively. The dark yellow dashed lines show the borders of the corresponding expected exclusion regions for the SM hypothesis. Exact Higgs alignment corresponds to  $\cos(\beta - \alpha) = 0$ . Taken from the ATLAS Collaboration, [JHEP 11 \(2024\) 097](#).

## My most recent Ph.D. students and their projects

- 2HDM theory and phenomenology (with **E. Shahly**). Received his Ph.D. in September 2025.
  - Neutral Higgs-mediated flavor violation in the lepton sector due to renormalization group running (with S. Gori and **E. Shahly**). Results to appear on the arXiv later this year.
  - One-loop renormalization of the 2HDM in the Higgs basis.
- Phenomenological aspects of more general 2HDMs (with **J.M. Connell**). Received his Ph.D. in June 2024.
  - Explored some (local)  $2-3\sigma$  deviations in LHC searches for new Higgs bosons, with implications for the flavor-aligned 2HDM. Results published in **Phys.Rev. D 108, 055031 (2023)**.
  - Examined the structure of lepton flavor-changing neutral currents mediated by neutral Higgs bosons in extended Higgs models. Results to appear on the arXiv later this year.

From a forthcoming paper in collaboration with Stefania Gori and [Eric Shahly](#). Off-diagonal couplings of the neutral Higgs boson to  $\tau\mu$  can be generated if flavor alignment is imposed at a very high energy scale  $\Lambda$ , due to renormalization group evolution from  $\Lambda$  down to the energy scale of electroweak physics (100 GeV).

## 4 Results

### 4.1 Lepton flavor violating decays of the SM-like Higgs boson

The partial widths for the decays of the SM-like Higgs field  $h$  into a pair of fermions are given below. Note that the color factor  $N_C = 3$  for quarks, and  $N_C = 1$  for leptons.

$$\begin{aligned} \Gamma(h \rightarrow f_i \bar{f}_i) = & \frac{N_C G_F}{4\sqrt{2}\pi} m_h m_{f_i}^2 \left[ \text{Re} \left( s_{\beta-\alpha} + \epsilon_6 c_{\beta-\alpha} \frac{\rho_f^{ii}}{\kappa_f^{ii}} \right)^2 \left( 1 - \frac{4m_{f_i}^2}{m_h^2} \right)^{3/2} \right. \\ & \left. + \text{Im} \left( s_{\beta-\alpha} + \epsilon_6 c_{\beta-\alpha} \frac{\rho_f^{ii}}{\kappa_f^{ii}} \right)^2 \left( 1 - \frac{4m_{f_i}^2}{m_h^2} \right)^{1/2} \right] \end{aligned} \quad (4.1)$$

$$\begin{aligned} \Gamma(h \rightarrow f_i \bar{f}_j) = \Gamma(h \rightarrow f_j \bar{f}_i) = & N_C \frac{m_h c_{\beta-\alpha}^2}{16\pi} (|\rho_f^{ij}|^2 + |\rho_f^{ji}|^2) \times \\ & \left[ 1 - \left( \frac{m_{f_i} - m_{f_j}}{m_h} \right)^2 \right] \times \left[ \left( 1 - \frac{m_{f_i}^2 + m_{f_j}^2}{m_h^2} \right)^2 - \frac{4m_{f_i}^2 m_{f_j}^2}{m_h^4} \right]^{1/2} \quad (i \neq j) \end{aligned} \quad (4.2)$$

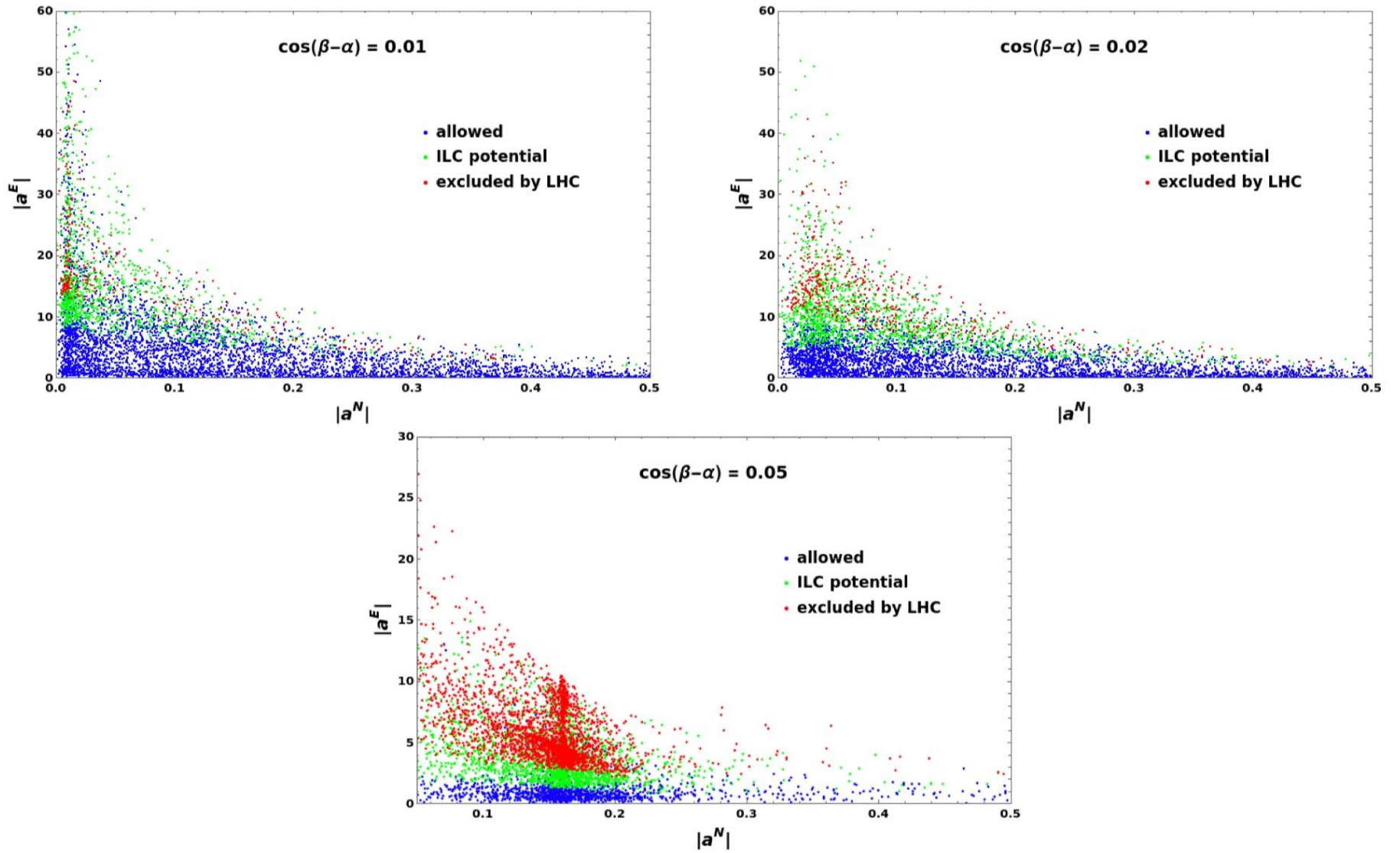


Figure 3:  $BR(h \rightarrow \mu\tau)$  results for the case of  $\cos(\beta - \alpha) = 0.01$  (left), 0.02 (right) and 0.05 (bottom) for fixed quark parameters  $a^U = 0.1$  and  $a^D = 1$ . Green points indicate choices of the alignment parameters that lead to  $h \rightarrow \mu\tau$  branching ratios that exceed the projected ILC upper bound of  $2.3 \times 10^{-4}$ , but are not yet excluded by LHC bounds. Red points are already excluded by LHC bounds and blue points remain unexcluded by both current experimental bounds and ILC projections.

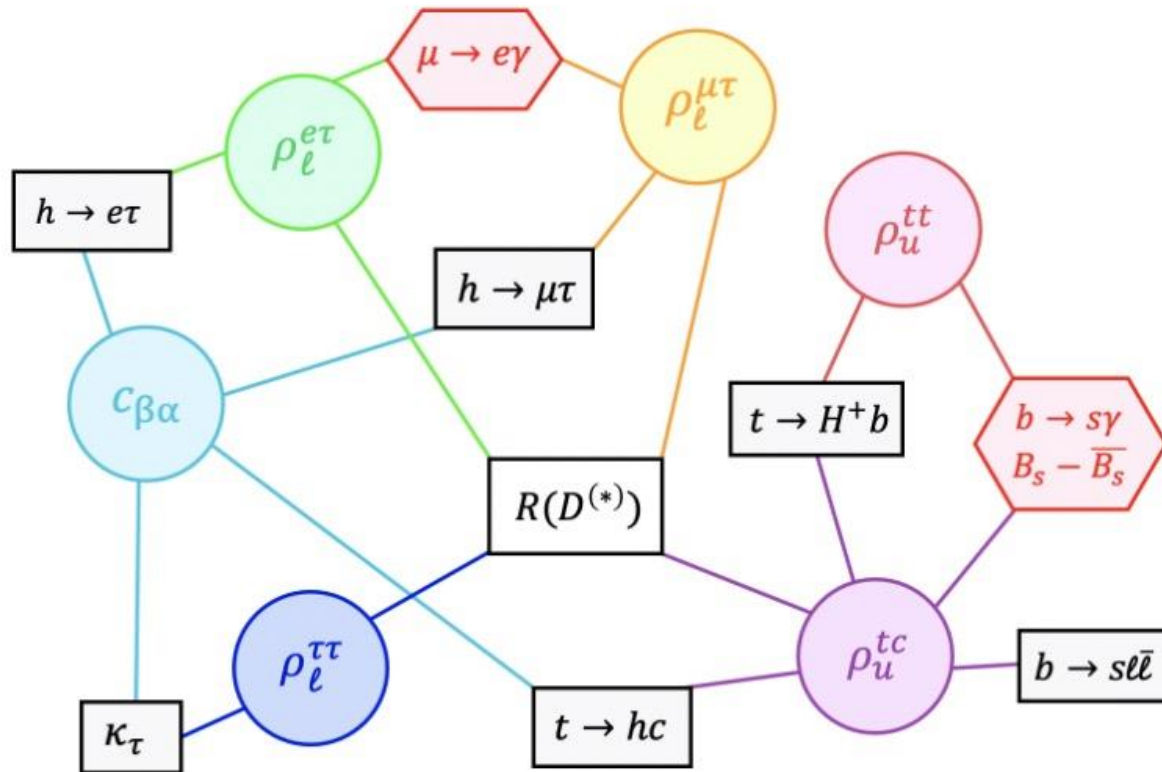


FIG. 3. Diagram showing the correlations between the free parameters (circles) of our model (except the Higgs masses) and the observables. Observables providing strong constraints are shown as red hexagons while the ones pointing towards a NP effect are shown as black rectangles.

Taken from A. Crivellin and S. Iguro, Phys. Rev. **D** 110, 015014 (2024).

From a forthcoming paper with **Joseph Connell**. Nondiagonal lepton—Higgs couplings are constrained by many observables. For example, consider  $\tau \rightarrow \mu \gamma$ .

$$A(\tau \rightarrow \mu \gamma) \simeq \frac{1}{16\pi^2} \left( \sqrt{2} \sum_{\phi} \frac{g_{\phi\mu\tau} g_{\phi\tau\tau}}{m_{\phi}^2} \left( \ln \frac{m_{\phi}^2}{m_{\tau}^2} - \frac{3}{2} \right) + 2 \sum_{\phi, f} g_{\phi\mu\tau} g_{\phi f f} \frac{N_c Q_f^2 \alpha}{\pi} \frac{1}{m_{\tau} m_f} f_{\phi} \left( \frac{m_f^2}{m_{\phi}^2} \right) \right. \\ \left. - \sum_{\phi=h, H} g_{\phi\mu\tau} C_{\phi WW} \frac{g\alpha}{2\pi m_{\tau} m_W} \left[ 3f_{\phi} \left( \frac{m_W^2}{m_{\phi}^2} \right) + \frac{23}{4} g \left( \frac{m_W^2}{m_{\phi}^2} \right) + \frac{3}{4} h \left( \frac{m_W^2}{m_{\phi}^2} \right) + m_{\phi}^2 \frac{f_{\phi} \left( \frac{m_W^2}{m_{\phi}^2} \right) - g \left( \frac{m_W^2}{m_{\phi}^2} \right)}{2m_W^2} \right] \right)$$

We define three integrals for real positive values of  $z$   $[1, 2]$ :

$$g(z) = \frac{1}{2} z \int_0^1 \frac{dx}{x(1-x) - z} \ln \left[ \frac{x(1-x)}{z} \right],$$

$$f(z) = \frac{1}{2} z \int_0^1 \frac{1 - 2x(1-x)}{x(1-x) - z} \ln \left[ \frac{x(1-x)}{z} \right] dx,$$

$$h(z) = -\frac{1}{2} z \int_0^1 \frac{dx}{x(1-x) - z} \left\{ 1 - \frac{z}{x(1-x) - z} \ln \left[ \frac{x(1-x)}{z} \right] \right\}.$$

Then, one can derive the following expressions for  $f(z)$  and  $h(z)$  in terms of  $g(z)$ :

$$f(z) = z(2 + \ln z) + (1 - 2z)g(z),$$

$$h(z) = \frac{z[2g(z) + \ln z]}{1 - 4z}.$$

An explicit expression for  $g(z)$  is given by:

$$g(z) = \begin{cases} \frac{z}{\sqrt{1-4z}} \left\{ \text{Li}_2(x_+) - \text{Li}_2(x_-) - \frac{1}{2} \ln z \ln \left( \frac{x_+}{x_-} \right) \right\}, & \text{for } 0 < z \leq \frac{1}{4}, \\ \frac{2z}{\sqrt{4z-1}} \text{Cl}_2 \left( 2 \sin^{-1} \frac{1}{2\sqrt{z}} \right), & \text{for } z > \frac{1}{4}, \end{cases} \quad (62)$$

where  $x_{\pm} \equiv \frac{1}{2} [1 \pm \sqrt{1-4z}]$  and  $0 \leq \sin^{-1}[1/(2\sqrt{z})] \leq \frac{1}{2}\pi$  (for  $z \geq \frac{1}{4}$ ). In Fig. 1, we have employed Mathematica (Version 14.0) to produce plots of the functions  $g(z)$ ,  $f(z)$  and  $-h(z)$  for  $0.01 \leq z \leq 100$ . This figure reproduces the results first shown in Fig. 3 of Ref. [2].

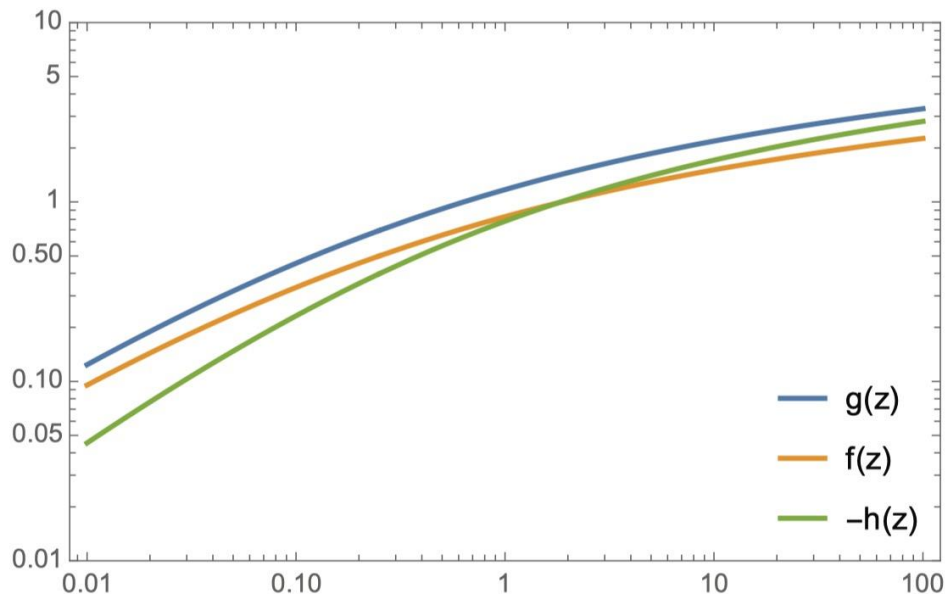


Figure 1: Plots of  $g(z)$  given by eq. (62),  $f(z)$  given by eq. (60), and  $h(z)$  given by eq. (61) as a function of the variable  $z$  for  $0.01 \leq z \leq 100$ . These plots were produced using Version 14.0 of Mathematica.

## Ongoing and Future Activities

- Reassessing the Cheng-Sher ansatz for off-diagonal flavor couplings of neutral Higgs bosons (with **Joseph Connell**).
- Basis-invariant treatment of the 3HDM (with V. Keus).
- Further studies of GOOFy “symmetries” and new RG-stable fixed points without a conventional symmetry explanation; extension to the Yukawa sector (with P. Ferreira)
- Beyond the  $S$ ,  $T$ , and  $U$  oblique parameters in extended electroweak models containing a dark  $Z$  boson (with J.P. Silva).
- The anapole moment of fundamental particles (with H. Dreiner).
- One-loop renormalization in the Higgs basis (with S. Kanemura).

# From Finite Groups to Lie Groups and Lie Algebras: A Guide for the Perplexed Physicist

HOWARD E. HABER AND JOHN TERNING

A new book is now being written, based on the Physics 251 course that I have taught many times during my time at UCSC.

I will be teaching this course again in the 2026 spring quarter. A rough draft of the book (currently at 600 pages) will be available to all students attending this course.

## Part I Theory of finite groups

<b>1 Introduction to Abstract Group Theory</b>	3
1.1 Basic Algebraic Structures	3
1.2 Further Algebraic Structures	11
1.3 Examples of Finite Groups	22
1.4 Examples of Infinite Groups	40
Exercises	42
<b>2 Fundamentals of Finite Groups</b>	47
2.1 Cosets	47
2.2 Equivalence Classes	50
2.3 Lagrange's Theorem	52
2.4 Normal Subgroups	52
2.5 Solvable and Nilpotent	59
2.6 Simple Groups and Monster Facts	60
2.7 Maps between Groups	60
2.8 Direct and Semidirect Products of Groups	63
2.9 Group Extensions	67
2.10 Conjugation	70
2.11 Groups of Fixed Order	72
2.12 Class Multiplication	79
2.13 Group Algebra	80
2.14 Invariant Mean on a Finite Group	83
2.15 A Taste of Galois Theory	84
Exercises	87
<b>3 Group Representation Theory</b>	90
3.1 Basics of Representations	90
3.2 Equivalent Representations	91
3.3 Reducible and Irreducible Representations	92
3.4 Unitary Representations	96
3.5 Schur's Lemmas	98
3.6 Great Orthogonality Theorem	100