DUE: WEDNESDAY DECEMBER 7, 2016

You must submit the final homework to my ISB mailbox by 6 pm on Wednesday December 7 in order to earn the proper credit. Solutions will then be posted on the class website.

1. Repeat the computation of problem 3 of Problem Set 4, but this time use the full relativistic expression for the matrix element. Show that the resulting spin-averaged differential cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4|\vec{\boldsymbol{p}}|^2\beta^2\sin^4(\theta/2)}\left(1 - \beta^2\sin^2\frac{\theta}{2}\right),$$

where $\vec{\boldsymbol{p}}$ is the three-momentum of the electron and $\beta \equiv v/c$ is the electron velocity. This is the famous Mott formula for the Coulomb scattering of relativistic electrons.

2. In class, the total cross section for $e^+e^- \to \mu^+\mu^-$ at very high center-of-mass energies ($\sqrt{s} \gg m_\mu$) was found to be $\sigma = 4\pi\alpha^2/(3s)$, where $\alpha \simeq 1/137$.

(a) Restore the appropriate factors of $\hbar$ and c in the formula for σ, so that you can evaluate σ in units of area.

(b) Find the value of σ in nanobarns (1 nb= 10^{-33} cm^2), for $\sqrt{s} = 10.58$ GeV (this is the center-of-mass energy of the B-factory at the KEKB collider in Tsukuba, Japan).

3. Compute the differential cross section, $d\sigma/d\Omega$, in the center-of-momentum frame, for Bhabha scattering, $e^+e^- \to e^+e^-$. You may assume that the total energy of the initial e^+e^- system in the center-of-momentum frame is much larger than the electron mass, m_e, in which case (to good approximation) you may set $m_e = 0$ in your calculation. There are two Feynman diagrams that contribute at second order in the perturbation series for the S-matrix. These two contributions must be added in the invariant matrix element before squaring. Be sure that you have the correct relative sign between these diagrams.

(a) After averaging over the initial state helicities and summing over the final state helicities, express the result for the squared invariant matrix element for Bhabha scattering in terms of the Mandelstam variables, s, t and u. Show that the differential cross-section can be cast into the following form,

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left[u^2\left(\frac{1}{s} + \frac{1}{t}\right)^2 + \left(\frac{t}{s}\right)^2 + \left(\frac{s}{t}\right)^2\right],$$

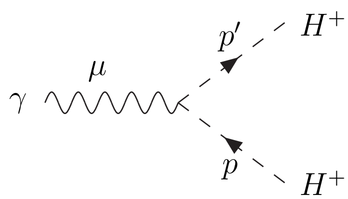
where $\alpha \equiv e^2/(4\pi)$. [*NOTE*: $s + t + u = 0$ in the approximation where $m_e = 0$.]

(b) Integrate over the azimuthal angle, and express $d\sigma/d\cos\theta$ as a function of the center-of-momentum scattering angle θ. Sketch the shape of the angular distribution of the scattered electron. What feature of the Feynman diagram is responsible for the divergence of $d\sigma/d\cos\theta$ as $\theta \to 0$?

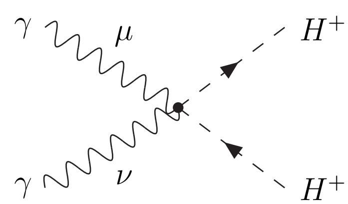
4. The interaction Lagrangian of scalar electrodynamics is:

$$\mathcal{L}_I = -ieA^\mu \left[H^-\partial_\mu H^+ - (\partial_\mu H^-)H^+\right] + e^2 A_\mu A^\mu H^+ H^- \,,$$

where e is the usual coupling strength of quantum electrodynamics. This Lagrangian describes the interaction of photons and charged scalars, $H^\pm$ (where $H^- = [H^+]^\dagger$). The Feynman rules for scalar electrodynamics are exhibited below.



$$-ie(p+p')^\mu$$



$$ie^2 C g^{\mu\nu}$$

The four-momenta (specified in the first Feynman rule above) point in the direction of the arrows as indicated.

(a) By explicitly computing the *first-order* S-matrix element for the scattering process $\gamma\gamma \to H^+H^-$, determine the number C that appears in the Feynman rule given above for the $\gamma\gamma H^+H^-$ vertex.

(b) At $\mathcal{O}(e^2)$, there are also contributions at second order to the S-matrix element. Using the above Feynman rules, write down the complete $\mathcal{O}(e^2)$ invariant matrix element for the scattering process $\gamma\gamma \to H^+H^-$. Simplify as much as possible each term that contributes to the matrix element by using the kinematical constraints of the problem.

(c) The matrix element obtained in part (b) takes the form $\mathcal{M} = \mathcal{M}_{\mu\nu}\,\epsilon_1^\mu(k_1)\epsilon_2^\nu(k_2)$, where k_1 and k_2 are the initial photon four-momenta and ϵ_1 and ϵ_2 are the corresponding photon polarization four-vectors. Verify that $k_1^\mu \mathcal{M}_{\mu\nu}\epsilon_2^\nu(k_2) = k_2^\nu \mathcal{M}_{\mu\nu}\epsilon^\mu(k_1) = 0$. That is, the replacement of either $\epsilon_1 \to k_1$ and/or $\epsilon_2 \to k_2$ in $\mathcal{M}$ yields an expression that vanishes. These relations are consequences of the gauge invariance of electrodynamics and provide an important check that all the contributing Feynman diagrams of $\mathcal{O}(e^2)$ have been correctly included. It also serves as a check of the value of C that you obtained in part (a).