$$\int \frac{d^nq}{(2\pi)^n}\,\frac{1}{(q^2+2q{\cdot}p-m^2+i\varepsilon)^r} = i(-1)^r(p^2+m^2)^{2-\epsilon-r}(4\pi)^{\epsilon-2}\,\frac{\Gamma(\epsilon+r-2)}{\Gamma(r)}$$

$$\int \frac{d^nq}{(2\pi)^n}\,\frac{q^\mu}{(q^2+2q{\cdot}p-m^2+i\varepsilon)^r} = -i(-1)^r(p^2+m^2)^{2-\epsilon-r}(4\pi)^{\epsilon-2}\,\frac{\Gamma(\epsilon+r-2)}{\Gamma(r)}\,p^\mu$$

$$\int \frac{d^nq}{(2\pi)^n}\,\frac{q^\mu q^\nu}{(q^2+2q{\cdot}p-m^2+i\varepsilon)^r} = i(-1)^r(p^2+m^2)^{2-\epsilon-r}(4\pi)^{\epsilon-2}\,\frac{\Gamma(\epsilon+r-3)}{\Gamma(r)} \times\left[(\epsilon+r-3)p^\mu p^\nu-\tfrac{1}{2}g^{\mu\nu}(p^2+m^2)\right]$$

$$\int \frac{d^nq}{(2\pi)^n}\,\frac{q^\mu q^\nu q^\alpha}{(q^2+2q{\cdot}p-m^2+i\varepsilon)^r} = -i(-1)^r(p^2+m^2)^{2-\epsilon-r}(4\pi)^{\epsilon-2}\,\frac{\Gamma(\epsilon+r-3)}{\Gamma(r)} \times\left[(\epsilon+r-3)p^\mu p^\nu p^\alpha-\tfrac{1}{2}(g^{\mu\nu}p^\alpha+g^{\mu\alpha}p^\nu+g^{\nu\alpha}p^\mu)(p^2+m^2)\right]$$

$$\begin{aligned}\int \frac{d^nq}{(2\pi)^n}\,\frac{q^\mu q^\nu q^\alpha q^\beta}{(q^2+2q{\cdot}p-m^2+i\varepsilon)^r} = {}& i(-1)^r(p^2+m^2)^{2-\epsilon-r}(4\pi)^{\epsilon-2}\,\frac{\Gamma(\epsilon+r-4)}{\Gamma(r)}\\ &\times\Big\{(\epsilon+r-3)(\epsilon+r-4)p^\mu p^\nu p^\alpha p^\beta-\tfrac{1}{2}(\epsilon+r-4)(g^{\mu\nu}p^\alpha p^\beta\\ &+g^{\mu\alpha}p^\nu p^\beta+g^{\mu\beta}p^\nu p^\alpha+g^{\nu\alpha}p^\mu p^\beta+g^{\nu\beta}p^\mu p^\alpha+g^{\alpha\beta}p^\mu p^\nu)(p^2+m^2)\\ &+\tfrac{1}{4}(g^{\mu\nu}g^{\alpha\beta}+g^{\mu\alpha}g^{\nu\beta}+g^{\mu\beta}g^{\nu\alpha})(p^2+m^2)^2\Big\}\end{aligned}$$

where $\varepsilon$ (not to be confused with $\epsilon$) is a positive infinitesimal constant and $\epsilon$ is related to the number of spacetime dimensions via

$$g^{\mu\nu}g_{\mu\nu}=n\equiv 4-2\epsilon\,.$$

In addition, we shall *define*

$$\int \frac{d^nq}{(2\pi)^n}\,\frac{1}{(q^2)^r}\equiv 0\,,$$

which corresponds to setting $p^2=m^2=0$ in the first integral above under the assumption that $\epsilon<2-r$. However, in the dimensional regularization procedure, we shall adopt the above definition for *all* $r$.

We can expand about $\epsilon=0$ by using

$$\Gamma(-N+\epsilon)=\frac{(-1)^N}{N!}\left[\frac{1}{\epsilon}+\psi(N+1)+\mathcal{O}(\epsilon)\right]\,,$$

where $N$ is a non-negative integer, $\psi(x) \equiv \Gamma'(x)/\Gamma(x)$ with $\Gamma'(x) \equiv d\Gamma(x)/dx$,

$$\psi(1) = -\gamma\,, \qquad\qquad \psi(N+1) = -\gamma + \sum_{k=1}^{N} \frac{1}{k}\,,$$

and $\gamma = -\Gamma'(1) = 0.5772\cdots$ is the Euler-Mascheroni constant. If the $\mathcal{O}(\epsilon)$ terms are needed, then one must use $x\Gamma(x) = \Gamma(x+1)$ until $\Gamma(1+\epsilon)$ is reached, and then use

$$\log\Gamma(1+\epsilon) = -\gamma\epsilon + \sum_{k=2}^{\infty} \frac{(-\epsilon)^k}{k}\,\zeta(k)\,,$$

where $\zeta(k)$ is the Riemann zeta function.

When fermions are involved, we need to consider the Dirac matrix algebra in $n$-dimensions. The Dirac gamma matrices are denoted by $\gamma^0$, $\gamma^1, \gamma^2, \ldots, \gamma^{n-1}$. The following $n$-dimensional gamma matrix relations must be used:

1. $\left\{\gamma^\mu\,,\,\gamma^\nu\right\} = 2g^{\mu\nu}$
2. $\gamma_\mu\gamma^\mu = n$
3. $\gamma_\mu\gamma^\alpha\gamma^\mu = (2-n)\gamma^\alpha$
4. $\gamma_\mu\gamma^\alpha\gamma^\beta\gamma^\mu = 4g^{\alpha\beta} + (n-4)\gamma^\alpha\gamma^\beta$
5. $\gamma_\mu\gamma^\alpha\gamma^\beta\gamma^\rho\gamma^\mu = -2\gamma^\rho\gamma^\beta\gamma^\alpha + (4-n)\gamma^\alpha\gamma^\beta\gamma^\rho$
6. $\gamma_\mu\gamma_\alpha\gamma_\beta\gamma_\rho\gamma_\sigma\gamma^\mu = 2(\gamma_\alpha\gamma_\sigma\gamma_\rho\gamma_\beta + \gamma_\beta\gamma_\rho\gamma_\sigma\gamma_\alpha) + (n-4)\gamma_\alpha\gamma_\beta\gamma_\rho\gamma_\sigma$
7. Trace formulae are unchanged. In particular, $\mathrm{Tr}\,\gamma^\mu\gamma^\nu = 4g^{\mu\nu}$, where the 4 in the trace formula is purely conventional. However, is crucial to use $g^{\mu\nu}g_{\mu\nu} = n$ in all calculations in $n$-dimensions before taking the $\epsilon \to 0$ limit.

The $n$-dimensional analog of $\gamma_5$, which satisfes $(\gamma_5)^2 = \mathbb{1}$ and $\mathrm{Tr}\ \gamma_5 = \mathrm{Tr}\ \gamma_5\gamma_\mu\gamma_\nu = 0$, is problematical since $\epsilon_{\alpha\beta\rho\sigma}$ is inherently four-dimensional. In some applications, one can take the anticommution relation, $\left\{\gamma^\mu\,,\,\gamma_5\right\} = 0$, to be valid for $n \neq 4$, but some care is required.

Finally, we record some of the Feynman parameter formulae:

$$\frac{1}{A^\alpha B^\beta} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 dx\, \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\big[xA + (1-x)B\big]^{\alpha+\beta}}$$

$$\frac{1}{A^\alpha B^\beta C^\delta} = \frac{\Gamma(\alpha+\beta+\delta)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\delta)} \int_0^1 x\,dx \int_0^1 dy\, \frac{x^{\alpha+\beta-2}\,y^{\alpha-1}(1-x)^{\delta-1}(1-y)^{\beta-1}}{\big[xyA + x(1-y)B + (1-x)C\big]^{\alpha+\beta+\delta}}$$

and more generally,

$$\frac{1}{A_1^{\alpha_1}A_2^{\alpha_2}\cdots A_N^{\alpha_N}} = \frac{\Gamma(\alpha_1+\alpha_2+\cdots+\alpha_N)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\cdots\Gamma(\alpha_N)} \int_0^1 dx_1 \int_0^1 dx_2 \cdots \int_0^1 dx_N\,\delta\left(\sum_{j=1}^{N} x_j - 1\right)$$
$$\times \frac{x_1^{\alpha_1-1}\,x_2^{\alpha_2-1}\cdots x_N^{\alpha_N-1}}{(x_1A_1 + x_2A_2 + \cdots + x_NA_N)^{\alpha_1+\alpha_2+\cdots+\alpha_N}}$$