

$$\rho(h)U\rho(g)^*U^{-1}\rho(h^{-1})=\rho(h)\rho(u(g))\rho(h^{-1})=\rho(hu(g)h^{-1})$$