

$$U\rho(g)U^\dagger = \rho(u(g)) = \begin{pmatrix} U^\dagger e^{-iT^aT\omega^a}U & 0 \\ 0 & Ue^{iT^a\omega^a}U^\dagger \end{pmatrix} = \begin{pmatrix} e^{iT^a\eta^a} & 0 \\ 0 & e^{-iT^aT\eta^a} \end{pmatrix}$$