

$$U^\dagger e^{-iT^a T \omega^a} U = e^{iT^a \eta^a} \Rightarrow U T e^{iT^a \omega^a} U^* = e^{-iT^a T \eta^a} = U e^{iT^a \omega^a} U^\dagger$$