Group Theory of Spontaneously Broken Gauge Symmetries

Qi Bin Lei

UC Santa Cruz/SCIPP

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Introduction

Overview of Today's Final Presentation

- Spontaneous Symmetry Breaking of $O(n)$ and $SU(n)$ [1] [2]
- Higgs Mechanism [3] [4] [5]
- Brief Standard Model Higgs [3]

Example: $O(3)$ (1)

- The Lagrangian for a triplet of scalar bosons with Yang-Mills fields, $\vec{A}_\mu$, is given by

$$\mathcal{L} = -\frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + \frac{1}{2}\left[\left(\partial_\mu - g\vec{t}\cdot\vec{A}_\mu\right)\vec{\phi}\right]^2 + \frac{1}{2}\mu^2\left(\vec{\phi}\cdot\vec{\phi}\right) - \frac{1}{4}\lambda\left(\vec{\phi}\cdot\vec{\phi}\right)^2 \qquad \text{(Li, 1.1)}$$

- Minimize potential and without loss of generality let the third component of the scalar field have a non-zero expectation value

$$\frac{\partial V}{\partial\phi_i} = \left(\mu^2 - \lambda\left(\vec{\phi}\cdot\vec{\phi}\right)\right)\phi_i = 0$$

$$\Rightarrow \langle 0|\,\phi_i\,|0\rangle = \delta_{i3}\,v \text{ and } v^2 = \frac{\mu^2}{\lambda}$$

- Redefine fields to have zero vacuum expectation value such that

$$\phi' = \phi_i - \left\langle\phi_i\right\rangle = \phi_i - \delta_{i3}\,v \Rightarrow \left\langle\phi'\right\rangle = 0$$

Example: $O(3)$ (2)

- Plugging in the newly redefined field into the Lagrangian results in,

$$\mathcal{L} = -\frac{1}{4}\left(\partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu\right)^2 + \frac{1}{2} g v^2 \left(A_{1\mu}^2 + A_{2\mu}^2\right) + \frac{1}{2}\left(\partial_\mu \vec{\phi}'\right)^2 + \left[\frac{1}{2}\left(\mu^2 - \lambda v^2\right)(\vec{\phi}' \cdot \vec{\phi}') - \lambda v^2 \phi_3'^2\right] + \left(\mu^2 - \lambda v^2\right) v \phi_3' \tag{Li, 1.3}$$

- Notice that $A_{\mu 1}$, $A_{\mu 2}$ get masses $g v^2$ and $A_{\mu 3}$ stays massless
- Applying $v^2 = \frac{\mu^2}{\lambda}$ results ϕ_1' and ϕ_2' being massless (which are also known as the Goldstone bosons)
- Note: Number of massive gauge bosons = Number of massless Goldstone bosons

Example: $O(3)$ (3)

- Taking a step back and looking at the potential,

$$\vec{\phi} \cdot \vec{\phi} = \frac{\mu^2}{\lambda}$$

- There are an infinite number of solution from by rotation in $O(3)$ space
- Since we choose $\phi_3 = \sqrt{\frac{\mu^2}{\lambda}}$, ϕ_1 and ϕ_2 are invariant under rotation
- So the symmetry is broken from $O(3)$ to $O(2)$

Generalized Symmetry Breaking Procedure

From Li,

1. Choose a particular representation for the scalar boson and write down the most general potential $V(\phi)$
2. Find the minimum of $V(\phi)$ by solving $\frac{\partial V}{\partial \phi} = 0$
3. Calculate the number of massless gauge bosons which determine the unbroken symmetry

Review: $O(n)$ Group

- Definition:

$$O(n) = \{R \in GL(n,\mathbb{R}) \mid R^T R = 1, \det R = \pm 1\}$$

- $O(n)$ has $\frac{1}{2}n(n-1)$ generators and is represented by

$$L_{ij} = X_i \frac{\partial}{\partial X_j} - X_j \frac{\partial}{\partial X_i},\ i, j = 1, \ldots, n$$

- The commutation relation among the generators is given by

$$\left[L_{ij}, L_{kl}\right] = \delta_{jk} L_{il} + \delta_{il} L_{jk} - \delta_{ik} L_{jl} - \delta_{jl} L_{ik}$$

- Irreducible representations in $O(n)$ can be classified as single-valued and double-valued representations
- Will be going over single-valued representations which transform as ordinary vectors and symmetrized or anti-symmetrized tensor products

$O(n)$ Group: Vector Representation (1)

- Most general potential and its minimum is,

$$
\begin{aligned}
V(\phi) &= -\frac{1}{2}\mu^2\phi_i\phi_i + \frac{1}{4}\lambda\left(\phi_i\phi_i\right)^2 \\
\Rightarrow \frac{\partial V}{\partial \phi_i} &= \left(-\mu^2 + \lambda\phi_j\phi_j\right)\phi_i = 0, \quad i = 1,\ldots,n \\
\Rightarrow |\vec{\phi}|^2 &= \phi_j\phi_j = \frac{\mu^2}{\lambda}
\end{aligned}
$$

- Similarly to $O(3)$ example, without loss of generality, choose $\vec{\phi} = \left(0, 0, \ldots, \left(\sqrt{\mu^2/\lambda}\right)\right)$
- Trivially, symmetry is broken from $O(n)$ to $O(n-1)$

$O(n)$ Group: Vector Representation (2)

- Lagrangian for the massive gauge bosons is,

$$\begin{aligned}\mathcal{L}_W &= \frac{1}{2} g^2 W^{\mu}_{ij} \left\langle \phi_j \right\rangle W^{\mu}_{ik} \langle \phi_k \rangle \\ &= \frac{1}{2} g^2 \sum_{i=1}^{n-1} \left(W^{\mu}_{in}\right)^2 \left(\mu^2/\lambda\right)\end{aligned} \tag{Li, 2.10}$$

- This explicitly shows that there are $(n-1)$ massive gauge bosons
- This implicitly shows that there are $\frac{1}{2}(n-1)(n-2)$ massless gauge bosons
- Having more sets of vector representations, say $|\vec{\phi}_1|, |\vec{\phi}_2|$ results in choosing corresponding nonzero components that result in $O(n)$ breaking to $O(n-2)$
- For electromagnetic interactions, if we were to model it with $O(n)$, must break $O(n)$ with $(n-2)$ sets of vectors in order to have $O(2) \cong U(1)$

$O(n)$ Group: Antisymmetric Second-Rank Tensor Representation (1)

- Most general potential is written as,

$$\begin{aligned} V(\phi) &= -\frac{1}{2}\mu^2\phi_{ij}\phi_{ij} + \frac{1}{4}\lambda_1\left(\phi_{ij}\phi_{ji}\right)^2 + \frac{1}{4}\lambda_2\left(\phi_{ij}\phi_{jk}\phi_{kl}\phi_{li}\right) \\ &= \frac{1}{2}\mu^2\,\mathrm{Tr}\,\phi^2 + \frac{1}{4}\lambda_1\left(\mathrm{Tr}\,\phi^2\right)^2 + \frac{1}{4}\lambda_2\,\mathrm{Tr}\,\phi^4 \qquad \text{(Li, 2.13)} \end{aligned}$$

- By the Canonical Form Theorem for Real Skew-Symmetric Matrices, we can always rewrite ϕ as a block diagonal

$$\phi = \begin{cases} \mathrm{diag}(A_1, \ldots, A_l), & n = 2l, \\ \mathrm{diag}(A_1, \ldots, A_l, 0), & n = 2l+1, \end{cases} \qquad A_i = a_i\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$O(n)$ Group: Antisymmetric Second-Rank Tensor Representation (2)

- The potential is then rewritten as,

$$V = -\mu^2 \sum_{i=1}^{l} a_i^2 + \lambda_1 \left(\sum_{i=1}^{l} a_i^2 \right)^2 + \frac{1}{2}\lambda_2 \left(\sum_{i=1}^{l} a_i^4 \right)$$

- Play our favorite game by minimizing the potential,

$$\frac{\partial V}{\partial a_i} = 2a_i \left[-\mu^2 + 2\lambda_1 \left(\sum_{j=1}^{l} a_j^2 \right) + \lambda_2 a_i^2 \right] = 0, \quad i = 1, \ldots, l$$

- Look for solutions where not all a_i's are zero for $i = 1, \ldots, k$, which results in

$$a_i^2 = \frac{\mu^2}{2\lambda_1 k + \lambda_2} \Rightarrow V = -\frac{k\mu^4}{2\lambda_1 k + \lambda_2} \qquad \text{(Li, 2.17)}$$

$O(n)$ Group: Antisymmetric Second-Rank Tensor Representation (3)

- Take derivative of potential with respect to k

$$\frac{dV}{dk} = -\frac{\mu^4 \lambda_2}{(2\lambda_1 k + \lambda_2)^2}$$

- If $\lambda_2 > 0$, then $\frac{dV}{dk} < 0$ so V decreases as k increases $\Rightarrow$ lowest energy requires largest possible k, let $k = l$

$$\phi = \begin{cases} \mathrm{diag}(A, \ldots, A), & n = 2l, \\ \mathrm{diag}(A, \ldots, A, 0), & n = 2l + 1, \end{cases} \qquad A = a\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad a = \sqrt{\left(\frac{\mu^2}{2l\lambda_1 + \lambda_2}\right)}$$

- Symmetry breaking pattern for both $O(2l)$ and $O(2l+1)$ is to $U(l)$

$O(n)$ Group: Antisymmetric Second-Rank Tensor Representation (4)

- Take derivative of potential with respect to k

$$\frac{dV}{dk} = -\frac{\mu^4 \lambda_2}{(2\lambda_1 k + \lambda_2)^2}$$

- If $\lambda_2 < 0$, then $\frac{dV}{dk} > 0$ so V increases as k increases $\Rightarrow$ lowest energy requires lowest possible k, let $k = 1$

$$\phi = \mathrm{diag}(B, 0, \ldots, 0), \qquad B = b\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad b = \sqrt{\left(\frac{\mu^2}{2\lambda_1 + \lambda_2}\right)}$$

- Symmetry breaking pattern is $O(n) \to U(1) \times O(n-2)$

Review: $SU(n)$ Group

- Definition:

$$SU(n) = \{U \in GL(n,\mathbb{C}) \mid U^\dagger U = 1,\ \det U = 1\}$$

- $SU(n)$ has $\left(n^2 - 1\right)$ generators, $U_i^j\ (i, j = 1, \ldots, n)$, with $U_j^i = \left(U_i^j\right)^\dagger$ and the commutation relation is given as,

$$\left[U_i^j, U_k^l\right] = \delta_j^k U_i^l - \delta_i^l U_k^j \qquad \text{(Li, 3.1)}$$

- Irreducible representations in $SU(n)$ are single-valued representations

$SU(n)$ Group: Vector Representation

- Most general potential and its minimum is,

$$\begin{aligned}
V(\psi) &= -\frac{1}{2}\mu^2\psi_i\psi_i + \frac{1}{4}\lambda\left(\psi_i\psi^i\right)^2 \\
&\Rightarrow \frac{\partial V}{\partial \psi} = \left(-\mu^2 + \lambda\psi_j\psi^j\right)\psi_i = 0, \quad i = 1,\ldots,n \\
&\Rightarrow \psi_j\psi^j = \frac{\mu^2}{\lambda}
\end{aligned}$$

- Similarly to $O(n)$, without loss of generality, choose $\langle\psi\rangle = \left(0, 0, \ldots, \left(\sqrt{\mu^2/\lambda}\right)\right)$
- Trivially, symmetry is broken from $SU(n)$ to $SU(n-1)$ which results in $(n-1)^2 - 1$ massless and $2n-1$ massive gauge bosons

$SU(n)$ Group: Symmetric Second-Rank Tensor Representation (1)

- Proceed as usual,

$$V(\psi) = -\frac{1}{2}\mu^2\psi_{ij}\psi^{ij} + \frac{1}{4}\lambda_1\left(\psi_{ij}\psi^{ij}\right)^2 + \frac{1}{4}\lambda_2\left(\psi_{ij}\psi^{jk}\psi_{kl}\psi^{li}\right) \qquad \text{(Li, 3.9)}$$

$$\frac{\partial V}{\partial\psi_{ij}} = -\frac{1}{2}\mu^2\psi^{ij} + \frac{1}{2}\lambda_1\left(\psi_{lm}\psi^{lm}\right)\psi^{ij} + \frac{1}{2}\lambda_2\left(\psi^{jk}\psi_{kl}\psi^{li}\right) = 0$$

- Let Hermitian matrix $X_l^k \equiv \psi_{lm}\psi^{mk}$ and rewrite the equation as,

$$\left[-\mu^2 + \lambda_1\left(\sum_{k=1}^{n} X_k\right) + \lambda_2 X_i\right]\psi_{ij} = 0, \quad j = 1,\ldots,n$$

- Follows same argument as $O(n)$ Antisymmetric Second-Rank Tensor Representation

$$X = c^2\mathbb{1} \Rightarrow c^2 = \frac{\mu^2}{\lambda_1 n + \lambda_2} \Rightarrow V(c) = -\frac{1}{4}\frac{n\mu^4}{\lambda_1 n + \lambda_2} \Rightarrow \frac{\partial V}{\partial n} = -\frac{\mu^4\lambda_2}{4\left(\lambda_1 n + \lambda_2\right)^2}$$

$SU(n)$ Group: Symmetric Second-Rank Tensor Representation (2)

- For $\lambda_2 > 0$, then $\frac{dV}{dn} < 0$ so V decreases as n increases $\Rightarrow$ lowest energy requires largest possible n

$$\psi = c\,\mathrm{diag}(1,1,\ldots,1)$$

- Symmetry is broken from $SU(n)$ to $O(n)$
- For $\lambda_2 < 0$, then $\frac{dV}{dn} > 0$ so V increases as n increases $\Rightarrow$ lowest energy requires lowest possible n

$$\psi = d\,\mathrm{diag}(1,0,\ldots,0), \quad d^2 = \frac{\mu^2}{\lambda_1 + \lambda_2}$$

- Symmetry is broken from $SU(n)$ to $SU(n-1)$

Summary of the Pattern of Symmetry Breaking

Table: Summary of the pattern of symmetry breaking.[1]

Representation	$O(n)$	$SU(n)$
vector	$O(n)$	$SU(n-1)$
k vectors	$O(n-k)$	$SU(n-k)$
2nd-rank symmetric tensor	$O(n-1)$	$SU(n-1)$
	or $O(l) \times O(n-l)$	or $O(n)$
2nd-rank antisymmetric tensor	$U(l)$	$O(2l+1)$
	or $U(1) \times O(n-2)$	or $SU(n-2) \times SU(2)$
adjoint representation		$SU(l) \times SU(n-l) \times U(1)$
		or $SU(n-1) \times U(1)$

[1] Corrections are made to $SU(n)$ anti-symmetric and adjoint representations [2]

Higgs Mechanism

Linear Sigma Model (1)

- Start with Lagrangian of a complex scalar field with a global $U(1)$ symmetry [2]

$$\mathcal{L} = \left(\partial_\mu \phi^*\right)\left(\partial_\mu \phi\right) + m^2 \phi\phi^* - \frac{\lambda}{4}\phi^2\phi^{*2} \qquad \text{(Schwartz, 28.10)}$$

- Minimize potential

$$\frac{\partial V(\phi)}{\partial \phi} = \left(m^2 - \frac{\lambda}{2}|\phi|^2\right)\phi^* = 0 \Rightarrow |\phi|^2 = \frac{2m^2}{\lambda} \Rightarrow v = \sqrt{\frac{2m^2}{\lambda}}$$

- Parametrize $\phi(x)$ around two real fields $\sigma(x)$ and $\pi(x)$ (and let F_π be a real number)as,

$$\phi(x) = \left(v + \frac{1}{\sqrt{2}}\sigma(x)\right) e^{i\frac{\pi(x)}{F_\pi}} \qquad \text{(Schwartz 28.11)}$$

[2] Apologies for the change in notation

Linear Sigma Model (2)

- The expanded out Lagrangian is,

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \sigma\right)^2 + \left(v + \frac{1}{\sqrt{2}}\sigma(x)\right)^2 \frac{1}{F_\pi^2}\left(\partial_\mu \pi\right)^2 \\ - \left(-\frac{m^4}{\lambda} + m^2\sigma^2 + \frac{1}{2}\sqrt{\lambda} m \sigma^3 + \frac{1}{16}\lambda\sigma^4\right) \qquad \text{(Schwartz 28.12)}$$

- π field is massless and is the Goldstone boson
- σ field is a massive particle whose potential comes from the radial direction of the Mexican hat potential
- U(1) symmetry is broken but is still realized with $\pi(x) \to \pi(x) + F_\pi \Theta$
- Note: Usually normalize kinetic π term with $F_\pi = \sqrt{2}v$

Abelian Higgs Model (1)

- Return back to Lagrangian from linear sigma model and gauge the $U(1)$ symmetry

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \left(\partial_\mu \phi^* - ieA_\mu \phi^*\right)\left(\partial_\mu \phi + ieA_\mu \phi\right) + m^2|\phi|^2 - \frac{\lambda}{4}|\phi|^4 \qquad \text{(Schwartz, 28.41)}$$

- Minimize potential and rewrite the $\phi(x)$ term as,

$$|\langle \phi \rangle| = \frac{v}{\sqrt{2}} = \sqrt{\frac{2m^2}{\lambda}}, \quad \phi(x) = \left(\frac{v + \sigma(x)}{\sqrt{2}}\right) e^{i\frac{\pi(x)}{F_\pi}} \qquad \text{(Schwartz, 28.42)}$$

- The Lagrangian then becomes,

$$\begin{aligned}\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 &+ \left(\frac{v+\sigma}{\sqrt{2}}\right)^2 \left[-i\frac{\partial \pi}{F_\pi} + \frac{\partial_\mu \sigma}{v+\sigma} - ieA_\mu\right]\left[i\frac{\partial_\mu \pi}{F_\pi} + \frac{\partial_\mu \sigma}{v+\sigma} + ieA_\mu\right] \\ &- \left(-\frac{m^4}{\lambda} + m^2\sigma^2 + \frac{1}{2}\sqrt{\lambda} m\sigma^3 + \frac{1}{16}\lambda\sigma^4\right) \qquad \text{(Schwartz, 28.43)}\end{aligned}$$

Abelian Higgs Model (2)

- Looking at the term involving A_μ^2 and σ^2

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}e^2 v^2 A_\mu^2 - m^2\sigma^2 \dots \qquad \text{(Schwartz, 28.44)}$$

- The gauge boson picks up as mass term $m_A = ev$
- The σ field has mass $m_\sigma = \sqrt{2}m$ and the π field is massless
- This Lagrangian obeys the gauge Symmetries

$$A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu \alpha(x), \quad \pi(x) \to \pi(x) - F_\pi \alpha(x) \qquad \text{(Schwartz, 28.47)}$$

Abelian Higgs Model (3)

Perform gauge fixing procedure

- Unitary gauge: $\pi(x) = 0$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \left(\frac{v+\sigma}{\sqrt{2}}\right)^2 \left[\frac{\partial_\mu \sigma}{v+\sigma} - ieA_\mu\right]\left[\frac{\partial_\mu \sigma}{v+\sigma} + ieA_\mu\right] - V(\sigma)$$

- Lorentz gauge: $\partial_\mu A_\mu = 0$ (mixing terms of A_μ and $\partial_\mu \sigma$ or $\partial_\mu \pi$ disappear)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}\left(\partial_\mu \sigma\right)^2 + \frac{1}{2}\left(\partial_\mu \pi\right)^2 + \frac{1}{2}e^2 v^2 A_\mu^2 - V(\sigma) + \text{interactions}$$

Abelian Higgs Model (4)

- For Unitary gauge, since we impose $\pi(x) = 0$, this results in $\alpha(x) = \frac{\pi(x)}{F_\pi}$ and ϕ loses its dependence on $\pi(x)$
- The potential then transforms as,

$$A_\mu \to A_\mu + \frac{1}{eF_\pi}\partial_\mu \pi(x)$$

- "Gauge boson eats the Goldstone boson through the Higgs mechanism"
- σ field is also known as the Higgs boson

Brief Standard Model Higgs (1)

- Symmetry breaking of $SU(2) \times U(1)_Y \to U(1)_{\text{EM}}$
- Start with a general Lagrangian of a complex $SU(2)$ doublet field (B_μ is the hypercharge gauge boson and W_μ^a are the $SU(2)$ gauge bosons)

$$\mathcal{L} = -\frac{1}{4}\left(W_{\mu\nu}^a\right)^2 - \frac{1}{4}B_{\mu\nu}^2 + \left(D_\mu H\right)^\dagger\left(D_\mu H\right) + m^2 H^\dagger H - \lambda\left(H^\dagger H\right)^2 \qquad \text{(Schwartz, 29.1)}$$

- Calculate minimum of potential and rewrite the H (where $\tau^a = \frac{1}{2}\sigma^2$)

$$v = \frac{m}{\sqrt{\lambda}}, \quad H = \exp\left(2i\frac{\pi^a\tau^a}{v}\right)\begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} \qquad \text{(Schwartz, 29.3)}$$

- The kinetic term then becomes,

$$|D_\mu H|^2 = g^2\frac{v^2}{8}\left[\left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2 + \left(\frac{g'}{g}B_\mu - W_\mu^3\right)^2\right] \qquad \text{(Schwartz, 29.4)}$$

Brief Standard Model Higgs (2)

- Interpreting the kinetic term,

$$|D_\mu H|^2 = g^2 \frac{v^2}{8}\left[\left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2 + \left(\frac{g'}{g}B_\mu - W_\mu^3\right)^2\right] \qquad \text{(Schwartz, 29.4)}$$

- The charged fields are,

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right) \Rightarrow \left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2 = 2W_\mu^+ W_\mu^- \Rightarrow \frac{g^2 v^2}{4} W_\mu^+ W_\mu^-$$

- The neutral field is,

$$Z_\mu = \frac{g W_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}} \Rightarrow \left(\frac{g'}{g}B_\mu - W_\mu^3\right)^2 = \frac{g^2 + g'^2}{g^2} Z_\mu Z_\mu \Rightarrow \frac{v^2}{8}\left(g + g'^2\right) Z_\mu Z_\mu$$

- Photon is $A_\mu = \frac{g W_\mu^3 + g' B_\mu}{\sqrt{g^2 + g'^2}}$ and doesn't appear $\Rightarrow$ massless!

Conclusion

- Spontaneous symmetry breaking of groups creates massive gauge bosons
- General procedure for Symmetry Breaking:
  - Write down most general Lagrangian and find the minimum of the potential
  - Find degrees of freedom/number of massless gauge bosons
- Higgs mechanism results in gauge fields combining with Goldstone bosons and resulting unbroken symmetries remain massless
- Extra degrees of freedom correspond to the Higgs boson

Thanks for listening! Any questions?

References

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Backup

(Li) Table 1: Properties of various representations in $O(n)$

Table: Properties of various representations in $O(n)$.

Representation	Dimension	Transformation law	Covariant derivatives $D_\mu\phi$
vector	n	$\phi_i \to \phi_i + \epsilon_{ij}\phi_j$	$\partial^\mu\phi_i - gW^\mu_{ij}\phi_j$
2nd-rank tensor sym	$\frac{1}{2}n(n+1)-1$	$\phi'_{ij} \to \phi'_{ij} + \epsilon_{ik}\phi'_{kj} + \epsilon_{jk}\phi'_{ik}$	$\partial^\mu\phi'_{ij} - gW^\mu_{ik}\phi'_{kj} - gW^\mu_{jk}\phi'_{ik}$
2nd-rank tensor asym	$\frac{1}{2}n(n-1)$	$\phi_{ij} \to \phi_{ij} + \epsilon_{ik}\phi_{kj} + \epsilon_{jk}\phi_{ik}$	$\partial^\mu\phi_{ij} - gW^\mu_{ik}\phi_{kj} - gW^\mu_{jk}\phi_{ik}$
spinor	2^l	$\chi_i \to \chi_i - \frac{1}{4}i\epsilon_{jk}(\sigma^{jk}\chi)_i$	$\partial^\mu\chi_i - \frac{1}{4}igW^\mu_{jk}(\sigma_{jk}\chi)_i$

(Li) Table 2: Properties of various representations in $SU(n)$

Table: Properties of various representations in $SU(n)$.

Representation	Dimension	Transformation law	Covariant derivative $D_\mu \phi$
vector	n	$\psi_i \to \psi_i + i\epsilon_i{}^j \psi_j$ $\psi^i = (\psi_i)^*,\ \epsilon_i{}^j = (\epsilon_j{}^i)^*$	$\partial_\mu \psi_i - igW_{\mu i}{}^j \psi_j$
2nd-rank tensor (symmetric)	$\frac{1}{2}n(n+1)$	$\psi_{ij} \to \psi_{ij} + i\epsilon_i{}^k \psi_{kj} + i\epsilon_j{}^k \psi_{ik}$ $\psi_{ij} = (\psi^{ij})^*,\ \psi_{ij} = \psi_{ji}$	$\partial_\mu \psi_{ij} - igW_{\mu i}{}^l \psi_{lj} - igW_{\mu j}{}^l \psi_{il}$
2nd-rank tensor (anti-symmetric)	$\frac{1}{2}n(n-1)$	$\psi_{ij} \to \psi_{ij} + i\epsilon_i{}^k \psi_{kj} + i\epsilon_j{}^k \psi_{ik}$ $\psi_{ij} = -\psi_{ji}$	$\partial_\mu \psi_{ij} - igW_{\mu i}{}^l \psi_{lj} - igW_{\mu j}{}^l \psi_{il}$
adjoint representation	$n^2 - 1$	$\psi_i{}^j \to \psi_i{}^j + i\epsilon_i{}^k \psi_k{}^j - i\epsilon_k{}^j \psi_i{}^k$ $\psi_i{}^j = (\psi_j{}^i)^*,\ \psi_i{}^i = 0$	$\partial_\mu \psi_i{}^j - igW_{\mu i}{}^l \psi_l{}^j + igW_{\mu l}{}^j \psi_i{}^k$

Goldstone Theorem

Goldstone Theorem: Every spontaneously broken continuous symmetry of a QFT gives rise to massless states. For an internal symmetry with breaking pattern $G \to H$ it leads to

$$\dim(G/H) = \dim(G) - \dim(H) = \dim(M_0)$$

massless particles: the Goldstone bosons. [4]

Higgs Mechanism

For a spontaneously broken gauge symmetry the transverse degrees of freedom of the broken symmetry gauge fields $A_\mu(x)$ combine with the Goldstone bosons $\xi(x)$ to complete the degrees of freedom of a massive vector field with the Goldstone modes becoming the longitudinal components. The gauge fields of the unbroken symmetries remain massless. The remaining scalar degrees of freedom are massive corresponding to the Higgs bosons. [4]