Physics 251 **Group Theory and Modern Physics** **Spring 2023**

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RECOMMENDED TEXTBOOKS:

A Physicist's Introduction to Algebraic Structures, by Palash B. Pal

Continuous Groups for Physicists, by Narasimhaiengar Mukunda and Subhash Chaturvedi

Textbooks used in previous Physics 251 classes:

Theory of Groups and Symmetries–Finite Groups, Lie Groups, and Lie Algebras, by Alexey P. Isaev and Valery A. Rubakov

Theory of Groups and Symmetries–Representations of Groups and Lie Algebras, Applications, by Alexey P. Isaev and Valery A. Rubakov

Group Theory in Physics: A Practioner's Guide, by Rutwig Campoamor-Stursberg and Michel Rausch de Traubenberg

Group Theory in Physics, by Wu-Ki Tung

Groups, Representations and Physics (2nd edition), by H.F. Jones

Group Theory: A Physicist's Survey, by Pierre Ramond

Group Theory for Physicists, by Zhong-Qi Ma

Lie Groups, Lie Algebras, and Some of Their Applications, by Robert Gilmore

Lie Algebras in Particle Physics (2nd edition), by Howard Georgi

Group Theory in a Nutshell for Physicists, by Anthony Zee

Additional outside reading:

Group Theory in Physics, Volume 1, by J.F. Cornwell

Group Theory in Physics, Volume 2, by J.F. Cornwell

Group Theory in Physics, Volume 3, by J.F. Cornwell

Problems and Solutions in Group Theory for Physicists, by Zhong-Qi Ma and Xiao-Yan Gu

Lie Groups and Lie Algebras for Physicists, by Ashok Das and Susumu Okubo

Symmetries, Lie Algebras and Representations, by Jürgen Fuchs and Christoph Schweigert

Symmetry Groups and Their Applications, by Willard Miller Jr.

Semi-Simple Lie Algebras and Their Representations, by Robert N. Cahn

Course Outline

1. Introduction to Abstract Group Theory
2. Fundamentals of Finite Groups
3. Group Representation Theory
4. The Symmetric Group and Young Tableaux
5. Introduction to Topological Groups and Lie Groups
6. SU(2) and SO(3)
7. Global and Local Properties of Lie Groups
8. Lie Algebras
9. Representations of SU(2) and SU(3)
10. Complex Semisimple Lie Algebras and their Representations

Course Requirements

The basic course requirements consist of four problem sets, which will be handed out during the quarter, and a term project. (There will be no exams.) Due to the limited time in a quarter, it will be impossible to do more than sketch some of the most basic applications of group theory to modern physics. To encourage students to delve deeper, all students will be required to complete a term project based on their reading of a particular topic in group theory and its applications to physics. The project may be presented orally or in written form at the end of the term. Oral presentations are encouraged since they will benefit all members of the class. Please follow the following schedule:

Initial choice of topic for term	May 2
Short written proposal for term	May 9
Oral Presentation of term project	June 14
Written version of term project	June 15

All projects should include a one page bibliography (containing references pertinent to the project). Copies of this bibliography should be made available to all students in the class. For those projects presented orally, a xerox of transparencies and a brief set of notes will be acceptable in lieu of a full written version.

I will be available during my office hours for suggestions and consultation on your choice for the term project. If you need some suggestions, you might consider choosing from the following list of possible topics for term projects.

Suggestions of topics for the term project

1. The Crystallographic Point Groups and Space Groups
2. Lattices, Bloch's Theorem and Band Theory
3. Group Theoretical Treatment of Vibrational Problems
4. Group Theory and Molecular Spectra
5. Group Theory and the Periodic Table
6. Group Theory in Classical Mechanics
7. Group Theoretic Methods in Quantum Mechanics
8. Group Theory in Condensed Matter Physics
9. Group Theory in General Relativity
10. Group Theory and the Shell Model in Nuclear Physics
11. Group Theory and the Quark Model in Particle Physics
12. Non-Abelian Discrete Symmetries in Particle Physics
13. Group Structure of Spontaneously Broken Gauge Theories
14. Group Theory and Grand Unification
15. Group Theory and Monopoles
16. The Lorentz and Poincaré Groups in Relativistic Field Theory
17. Applications of Clifford Algebras (and spinors) in physics
18. Boson and Fermion Realizations of Lie algebras
19. Spectrum generating groups (a.k.a. noninvariance groups)
20. Coherent states as a problem in group theory

For inspiration, in addition to the recommended outside reading and the many references on group theory for physicists provided in the bibliography that follows, have a look at:

Symmetry, Broken Symmetry, and Topology in Modern Physics–A First Course, by Mike Guidry and Yang Sun

Quantum Theory, Groups and Representations: An Introduction, by Peter Woit
(final draft version available from: `https://www.math.columbia.edu/~woit/QM/qmbook.pdf`)

Classical and Quantum Mechanics via Lie algebras, by Arnold Neumaier and Dennis Westra
(draft version available from: `https://arxiv.org/pdf/0810.1019v2.pdf`)

Bibliography

Other selected references in group theory for physicists:

Lectures on Group Theory for Physicists, by A.P. Balachandran and C.G. Trahern

Lie Algebras Part 1: Finite and Infinite Dimensional Lie Algebras and Applications in Physics, by G.G.A. Bäuerle and E.A. de Kerf

Lie Groups and Lie Algebras: A Physicist's Perspective, by Adam M. Bincer

Group Theory and General Relativity, by Moshe Carmeli

Symmetries in Quantum Mechanics: From Angular Momentum to Supersymmetry, by M. Chaichian and R. Hagedorn

Group Theory: Birdtracks, Lie's, and Exceptional Groups, by Predrag Cvitanović

Group Theory: Application to the Physics of Condensed Matter, by Mildred S. Dresselhaus, Gene Dresselhaus and Ado Jorio

Symmetries and Condensed Matter Physics, by Michael El-Batanouny and Frederick Wooten

Group Theoretical Methods and Their Applications, by A. Fässler and E. Stiefel

Groups and Manifods: Lectures for Physicists with Examples in Mathematica, by Pietro Guiseppe Fré and Alexander Fedotov

Lie Groups, Physics and Geometry, by Robert Gilmore

Quantum Mechanics: Symmetries, by Walter Greiner and Berndt Müller

Symmetries and Curvature Structure in General Relativity, by G.S. Hall

Group Theory and its Application to Physical Problems, by Morton Hammermesh

Lie Algebras and Applications (2nd edition), by Francesco Iachello

Lectures on Groups and Vector Spaces for Physicists, by Chris J. Isham

An Introduction to Tensors and Group Theory for Physicists (2nd edition), bu Nadir Jeevanjee

Theory of Groups in Classical and Quantum Physics, by Théo Kahan

Primer for Point and Space Groups, by Richard L. Liboff

Symmetries in Physics: Group Theory Applied to Physical Problems, by W. Ludwig and C. Falter

Lectures on Advanced Mathematical Methods for Physicists, by Sunil Mukhi and N. Mukunda

Generalized Coherent States and their Applications, by A. Perelomov

Lie Groups and Algebras with Applications to Physics, Geometry and Mechanics, by D.H. Sattinger and O.L. Weaver

A Course in the Application of Group Theory to Quantum Mechanics, by I.V. Schensted

Physics from Symmetry, by Jakob Schwichtenberg

Relativity, Groups, Particles, by Roman U. Sexl and Helmuth K. Urbantke

Classical and Quantum Mechanics with Lie Algebras, by Yair Shapira

Group Theory and Physics, by S. Sternberg

Shattered Symmetry, by Pieter Thyssen and Arnout Ceulemans

Group Theory and Quantum Mechanics, by Michael Tinkham

Group Theory and its Applications to the Quantum Mechanics of Atomic Spectra, by E. Wigner

Classical Groups for Physicists, by Brian G. Wybourne

Selected references in topology and differential geometry for physicists:

Differential Geometry and Lie Groups for Physicists, by Marián Fecko
The Geometry of Physics: An Introduction (3rd edition), by Theodore Frankel
Differential Geometry and Lie Groups–A Computational Perspective, by Jean Gallier and Jocelyn Quaintance
Differential Geometry and Lie Groups–A Second Course, by Jean Gallier and Jocelyn Quaintance
Fundamental Groups and Covering Spaces, by Elon Lages Lima
Topology: An Introduction with Application to Topological Groups, by George McCarty
A First Course in Topology: Continuity and Dimension, by John McCleary
Geometry, Topology and Physics (2nd edition), by M. Nakahara
Geometry and Topology, by Miles Reid and Balázs Szendrői
Geometrical Methods of Mathematical Physics, by Bernard Schutz

Other selected mathematical references:

Basic Lie Theory, by Hossein Abbaspur and Martin Moskowitz
Conformal Groups in Geometry and Spin Structures, by Pierre Anglès
An Introduction to Lie Groups and the Geometry of Homogeneous Spaces, by A. Arvanitoyeorgos
Matrix Groups: An introduction to Lie group theory, by Andrew Baker
Theory of Group Representations and Applications, by A.O. Barut and R. Raczka
Representations of Compact Lie Groups, by Theodor Bröcker and Tammo tom Dieck
Lie Groups (2nd edition), by Daniel Bump
Lectures on Lie Groups and Lie Algebras, by R.W. Carter, G. Segal and I.G. Macdonald
Lie Groups, by P.M. Cohn
Matrix Groups, by Morton L. Curtis
The Geometry of Octonians, by Tevian Dray and Corinne A. Manogue
Transformation Groups for Beginners, by S.V. Duzhin and B.D. Chebotarevsky
Clifford Algebras: An Introduction, by D.J.H. Garling
Representations of the Rotation and Lorentz Groups and their Applications, by I.M. Gel'fand, R.A. Minlos and Z. Ya. Shapiro
Basics of Lie Groups, by Michel Gourdin
Lie Algebras: Theory and Algorithms, by W.A. De Graaf
Continuous Groups of Transformations, by Luther Eisenhart
Introduction to Lie Algebras, by Karin Erdmann and Mark Wildon
Lie Groups, Lie Algebras, and Representations (2nd edition), by Brian C. Hall
Spinors and Calibrations, by F. Reese Harvey
Lie Groups, Lie Algebras, by Melvin Hausner and Jacob Schwartz
The Structure and Geometry of Lie Groups, by Joachim Hilgert and Karl-Hermann Neeb

Lectures on Representation Theory, by Jing-Song Huang
Introduction to Lie Algebras and Representation Theory, by James E. Humphreys
Lie Algebras, by Nathan Jacobson
An Introduction to Lie Groups and Lie Algebras, by Alexander Kirillov, Jr.
Lie Groups Beyond an Introduction (2nd edition), by Anthony W. Knapp
Groups and Symmetries: From Finite Groups to Lie Groups (2nd edition), by Yvette Kosmann-Schwarzbach
Theory of Continuous Groups, by Charles Loewner
Clifford Algebras and Spinors, by Pertti Lounesto
Topological Groups: An Introduction, by Nelson G. Markley
Lie Theory and Special Functions, by Willard Miller, Jr.
The Unitary and Rotation Groups, by Frank Murnaghan
Linear Representations of the Lorentz Group, by M.A. Naimark
Theory of Group Representations, by M.A. Naimark and A.I. Stern
Lectures on Real Semisimple Lie Algebras and Their Representations, by Arkady L. Onishchik
Lie Groups and Algebraic Groups, by Arkady L. Onishchik and Ernest B. Vinberg
The Lie Algebras su(N)—An Introduction, by Walter Pfeifer
Topological Groups, by L.S. Pontryagin
Clifford Algebras and the Classical Groups, by Ian R. Porteous
Lie Groups: A Problem-Oriented Introduction via Matrix Groups, by Harriet Pollatsek
Topological Groups, by L.S. Pontryagin
Lectures in Geometry, Semester V: Lie Groups and Lie Algebras, by M. Postnikov
Lie Groups: An Approach through Invariants and Representations, by Claudio Procesi
Geometry of Lie Groups, by Boris Rosenfeld
Lie Groups, An Introduction Through Linear Groups, by Wulf Rossmann
An Introduction to the Theory of Groups (4th edition), by Joseph J. Rotman
The Symmetric Group, by Bruce E. Sagan
Introduction to Lie Groups and Lie Algebras, by Arthur A. Sagle and Ralph E. Walde
Representing Finite Groups–A Semisimple Introduction, by Ambar N. Sengupta
Compact Lie Groups, by Mark R. Sepanski
Representations of Finite and Compact Groups, by Barry Simon
Naive Lie Theory, by John Stillwell
Special Functions: A Group Theoretical Approach, by James D. Talman
Matrix Groups for Undergraduates (2nd edition), by Kristopher Tapp
An Introduction to Clifford Algebras and Spinors, by Jayme Vaz, Jr. and Roldão Da Rocha, Jr.
Group and Representation Theory, by J.D. Vergados
Lie Algebras, by Zhe-Xian Wan
Compact Lie Groups and their Representations, by D.P. Zelobenko

Other selected references in group theory for particle physicists:

Group Theory in Particle, Nuclear, and Hadron Physics, by Syed Afsar Abbas

Lectures on Group Theory and Particle Theory, by H. Bacry

Group Theory for the Standard Model of Particle Physics and Beyond, by Ken J. Barnes

Symmetries and Group Theory in Particle Physics: An Introduction to Space-Time and Internal Symmetries, by Giovanni Costa and Gianluigi Fogli

Mathematical Gauge Theory: With Applications to the Standard Model of Particle Physics, by Mark J.D. Hamilton

Symmetries And Conservation Laws In Particle Physics: An Introduction To Group Theory for Particle Physicists, by Stephen Haywood

An Introduction to Non-Abelian Discrete Symmetries for Particle Physicists, by H. Ishimori et al.

Geometry, Symmetries, and Classical Physics–A Mosaic, by Manousos Markoutsakis

Group Structure of Gauge Theories, by L. O'Raifeartaigh

Symmetry and the Standard Model: Mathematics and Particle Physics, by Matthew Robinson

Group Theory for High Energy Physicists , by Mohammad Saleem and Muhammad Rafique

Symmetries in Fundamental Physics (2nd edition), by Kurt Sundermeyer

Group Theory in Subnuclear Physics, by Fl. Stancu

Tables of Lie Group and Lie Algebra information

Tables of Dimensions, Indices, and Branching Rules for Representations of Simple Lie Algebras, by W. G. McKay and J. Patera

Group Theory for Unified Model Building, *Physics Reports* **79** (1981) 1–128, by Richard Slansky

Finite-Dimensional Lie Algebras and Their Representations for Unified Model Building, arXiv:1511.08771v2, by N. Yamatsu

LieART 2.0–A Mathematica application for Lie Algebras and Representation Theory, Comput. Phys. Commun. 257, 107490 (2020), Robert Feger, Thomas W. Kephart, and Robert J. Saskowski