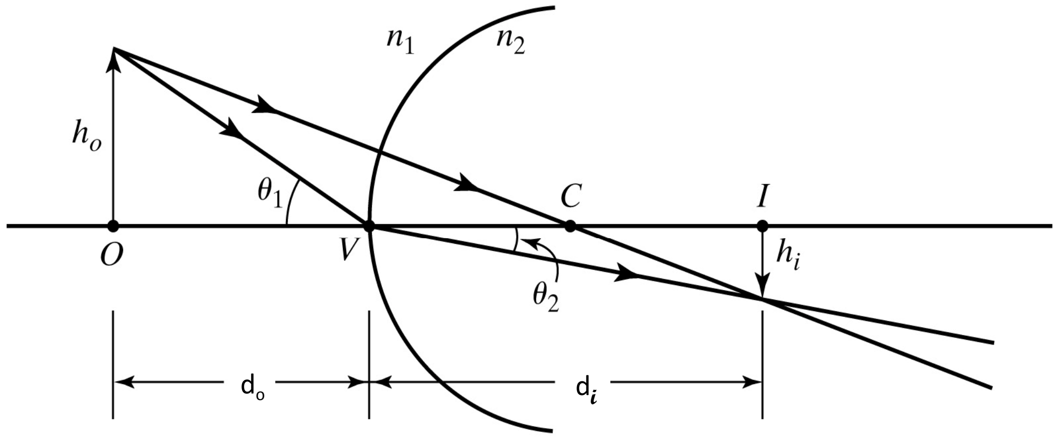
Lateral magnification of an image formed by refraction at a spherical surface

Although Giancoli provides a formula for determining the location of an image formed by refraction at a spherical surface [cf. Eq. 32-8], he does not compute the lateral magnification of the image. In fact, the computation is quite simple.



Referring to the figure above, the object is denoted by O, the image is denoted by I, the center of curvature of the spherical surface is denoted by C and the symmetry axis from O to I crosses the spherical surface at V.

For paraxial rays, we can assume that the angles $\theta_1 \ll 1$ and $\theta_2 \ll 1$. Hence, in the small angle approximation,

$$\theta_1 \simeq \frac{h_o}{d_o}\,, \qquad\qquad \theta_2 \simeq \frac{-h_i}{d_i}\,,$$

where h_o is the height of the object, h_i is the height of the image, and d_o and d_i are the object and image distances, respectively. We have inserted a minus sign in h_i, since $h_i < 0$ if the image is inverted as depicted in the figure, whereas θ_2 is a positive angle as shown. Using Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$, which in the small angle approximation reads $n_1\theta_1 \simeq n_2\theta_2$. Thus, using the results above for θ_1 and θ_2,

$$\frac{n_1 h_o}{d_o} \simeq -\frac{n_2 h_i}{d_i}\,.$$

Hence, the lateral magnification, which is defined by $m \equiv h_i/h_o$ is given by:

$$\boxed{m = \frac{h_i}{h_o} = -\frac{n_1 d_i}{n_2 d_o}\,.}$$